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Anisotropic Bianchi type VI_0 cosmological Model in Self-Creation Theory of Gravitation

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Abstract

In this paper, Anisotropic Bianchi VI_0 cosmological model is investigated in the presence of perfect fluid coupled with electromagnetic field in Self–Creation Theory of Gravitation. The physical condition that is shear scalar is directly proportional to the expansion scalar is used to obtain the solution of the field equations. The physical and geometrical properties of the obtained model are discussed. Also, we study the thermodynamic quantities (Entropy, Enthalpy, Gibbs energy and Helmholtz energy) of the universe with help of the scalar field ϕ. In addition to this, we studied the Statefinder diagnostic pair and jerk parameter of the universe.

Keywords: Bianchi type VI_0; Self Creation theory; Perfect fluid; Electromagnetic field

1. Introduction

Einstein introduced his theory of relativity in 1917. However, it faced several criticisms because the general theory of relativity could not fully account for Mach's principal and the equivalence principal. To address these shortcomings, scalar-tensor theories were developed. In 1982, Barber [1] proposed two self –creation cosmological models by modifying Brans and Dicke's theory [2] and general relativity. The first was a revised version of the Brans-Dicke theory, which proved unsatisfactory as it violated the equivalence principle. The second was a modification in general relativity into a variable G-theory, where the scalar field ϕ does not directly produce gravitation but instead acts as the reciprocal of the gravitational constant by dividing the matter tensor. In this model, the scalar field interacts with the trace of the energy-momentum tensor. Later, in 2010, Barber [3] revisited and reviewed his self-creation theory.

In recent years, self-creation theory has been the focus of many authors, who have used different space- time models with different energy-momentum tensors. Hadole et al. [4-5], Nimkar et al. [6], Katore et al. [7], Santhi et al.[8], Nimkar et al. [9], Advani [10], Jain et al. [11], Vinutha et al.[12], Tade et al.[13], Sen [14], Rao et al.[15], Pawar et al.[16-17], Reddy et al. [18], Chirde et al.[19], Tiwari et al. [20], Jaiswal et al. [21], Mahanta et al. [22-23], Rao et al. [24], Rai et al.[25], Katore et al.[26-27], Rao et al.[28], Reddy et al.[29], Adhav et al.[30-32], Singh et al.[33-34], Reddy et al.[35], Shanthi et al.[36], Mohanty et al.[37] are some of the researchers who have investigated the various aspects of Barbers second self – Creation theory.

The Barber field equation in the second self- creation theory can be expressed as

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$$R_{ij} - \frac{1}{2} Rg_{ij} = -8\pi\phi^{-1}T_{ij} \tag{1}$$

And

$$\phi = \varphi^{'k}_{;k} = \frac{8\pi\lambda}{3} T \tag{2}$$

Where ϕ the Barber is Scalar, T_{ij} is the energy- momentum tensor, is the invariant D'Alembertian, T is the trace of energy-monentum tensor T_{ij} and λ is a coupling constant. In the $\lim \lambda \to 0$, this theory approaches Einstein's general theory of relativity in every respect.

There are a few studies I the literature about Electromagnetic field. Mapari et al. [38] have investigated Interacting field cosmological model in Lyra geometry. Mete et al. [39] have obtained Magnetized plane symmetric cosmological model with wet dark fluid in scalar tensor theory of gravitation. Bhoyar et al.[40] Studied Magnetized Anti-stiff fluid cosmological models with variable cosmological constant. Katore et al.[41] have obtained Magnetized cosmological models in Saez Ballester theory of Gravitation. Deo et al. [42] have analyzed wet dark fluid in Bianchi type VI_0 universe with electromagnetic field. Patil et al. [43] Thick Domain walls coupled with viscous fluid and electromagnetic field in lyra geometry. Sahoo et al.[44] have discussed Bianchi type V and VI_0 cosmic strings coupled with Maxwell Fields in Bimetric theory. and recently Ugale et al.[45] have discussed Anisotropic bianchi type VI_0 cosmological models in a modified $f(R,T)$ gravity. Nimkar et al. [46] studied Anisotropic Bianchi type- VI_0 wet dark fluid cosmological models in Lyra's manifold.

In addition to this, wath et al.[47], Dhore et al. [48], Koussour et al.[49], Hegazy et al.[50-54], Nojiri et al.[55], Ayala et al.[56], Hawking et al.[57], Santos et al.[58], Prigogine et al.[59],Maity et al.[60], are some of the authors who have investigated different aspects of Thermodynamics.

Building on the above research, we develop a Bianchi type VI_0 cosmological model within the framework of self-creation theory, incorporating a perfect fluid coupled with an electromagnetic field. In section 2, the corresponding field equations for this model are formulated. Section 3 presents the solutions of this field equations, along with an analysis of the model's physical and kinematical properties and various cosmological l parameters. The influence of the perfect fluid coupled with the electromagnetic field on the universe is further examined in Section 4 through thermodynamics functions such as entropy, enthalpy, Helmoheltz energy and Gibbs free energy. Finally, Section 5 provides the conclusions drawn from the study.

2. Metric and Field Equation

The line element for the Bianchi type VI_0 cosmological model can be written in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}dy^2 + c^2e^{-2x}dz^2 \tag{3}$$

Where A, B, C are metric potential and function of t- only.

We consider the energy-momentum tensor to represent an interacting field and analyze the properties of the cosmological model within the framework of a coexisting system composed of a linearly coupled perfect fluid distribution and a source representing a free electromagnetic field, which is expressed as

$$T_{ij} = S_{ij} + E_{ij} \tag{4}$$

where S_{ij} represents the energy source for perfect fluid distribution and E_{ij} represents the electromagnetic energy momentum tensor and it is given by,

$$S_{ij} = (\rho + p)u_i u_j + p g_{ij} \tag{5}$$

together with

$$g_{ij} u_i u_j = -1 \tag{6}$$

u^i is the four velocity vector of the fluid , p and ρ are the proper pressure and energy density respectively.

$$E_{ij} = \frac{1}{4\pi}\left[g^{\alpha p} F_{i\alpha} F_{j\beta} - \frac{1}{4} g_{ij} F_{\alpha\beta} F^{\alpha\beta} \right] \tag{7}$$

Here E_{ij} is that for electromagnetic field and F_{ij} is the Maxwell's electromagnetic tensor. In a co-moving transformation system the magnetic field is considered along x direction so that only non- vanishing components of the electromagnetic field tensor F_{ij} are only F_{23} and F_{32}. Also, we have an electromagnetic field tensor that is anti-symmetric. The first set of Maxwell's equation

$$F_{[ij;k]} = 0 \quad \text{i.e.} \quad \left(F^{ij} \sqrt{-g} \right)_{;j} = 0 \tag{8}$$

Therefore equation (8) gives, F_{23} = constant = D (9)

Now from equation (5) and equation (7) for the metric equation (3) we have,

$$S_1^1 = S_2^2 = S_3^3 = p \quad ; \; S_4^4 = -\rho \tag{10}$$

$$E_1^1 = -E_2^2 = -E_3^3 = E_4^4 = -\frac{D^2}{8\pi B^2 C^2} \tag{11}$$

By using equation (10) and equation (11) in equation (1) of self-creation theory can be reduced for equation (3) as follows,

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = -8\pi\phi^{-1}\left[p - \frac{D^2}{8\pi B^2 C^2} \right] \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -8\pi\phi^{-1}\left[p + \frac{D^2}{8\pi B^2 C^2} \right] \tag{13}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -8\pi\phi^{-1}\left[p + \frac{D^2}{8\pi B^2 C^2} \right] \tag{14}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} = -8\pi\phi^{-1}\left[-\rho - \frac{D^2}{8\pi B^2 C^2} \right] \tag{15}$$

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \tag{16}$$

$$\ddot{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{\phi}=\frac{8\pi\lambda}{3}T \tag{17}$$

Where an overhead dot represents ordinary differentiation with respect to time t

We define the following physical parameters for solving the above field equation. The average scale factor $a(t)$ and spatial volume V are given by

$$a(t)=\left(AB^{2}\right)^{\frac{1}{3}} \tag{18}$$

$$V=a(t)^{3}=AB^{2} \tag{19}$$

The mean generalized Hubble's parameter H for this model is given by

$$H=\frac{1}{3}\left(H_x+H_y+H_z\right)=\frac{1}{3}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) \tag{20}$$

where, H_x, H_y and H_z are the directional Hubble parameters along x, y and z axis respectively defined by $H_x=\frac{\dot{A}}{A}$, $H_y=H_z=\frac{\dot{B}}{B}$

The average anisotropic parameter A_m for the universe is defined as

$$A_m=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \qquad \text{where,} \quad \Delta H_i=H_i-H \tag{21}$$

The anisotropic parameter serves as an indicator to assess whether the universe is undergoing anisotropic (direction-dependent) or isotropic (direction-independent) expansion. A nonzero value of t is parameter implies that the expansion rates differ along different spatial directions (anisotropic), whereas a value approaching zero indicates that the expansion is the same in all direction (isotropic). The universe expands anisotropically if $A_m \neq 0$ and it expands isotropically if $A_m=0$.

The expansion scalar θ and shear scalar σ^2 are given by

$$\theta=3H=\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) \tag{22}$$

$$\sigma^2=\frac{3}{2}A_m H^2=\frac{1}{3}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)^2 \tag{23}$$

3. Solution of Field Equations

Integrating equation (16), we get

$$B = \mu C \tag{24}$$

Where μ is constant of integration.

Here, we put $\mu = 1$ in eq. (24) we get $B = C$ and then substituting this in equation (12) to equation (15) we get the field equation are as follow

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{A^2} = -8\pi\phi^{-1}\left[p - \frac{D^2}{8\pi B^4}\right] \tag{25}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -8\pi\phi^{-1}\left[p + \frac{D^2}{8\pi B^4}\right] \tag{26}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = -8\pi\phi^{-1}\left[p + \frac{D^2}{8\pi B^4}\right] \tag{27}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = -8\pi\phi^{-1}\left[-\rho - \frac{D^2}{8\pi B^4}\right] \tag{28}$$

$$\ddot{\phi}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{\phi} = \frac{8\pi\lambda}{3}(3p - \rho) \tag{29}$$

Equations (25) to equation (28) are a system of four equations in five unknowns A, B, ϕ, p, ρ to obtained the determinate solution we take the help of the shear scalar σ is proportional to scalar expansion θ.

$$\sigma \propto \theta \Rightarrow \sigma = k_1\theta \tag{30}$$

where k_1 is a proportionality constant. Using equation (22) and (23) in equation (30) becomes,

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = k_1\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{31}$$

on integrating equation (31) we get,

$$A = B^{\left(\frac{2\sqrt{3}k_1 + 1}{1 - \sqrt{3}k_1}\right)} \tag{32}$$

Also, consider a generalized linearly varying deceleration parameter (Akarsu and Dereli) [61]

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = -k_2 t + n - 1 \tag{33}$$

Where $k_2 \geq 0$ and $n \geq 0$ are constants and $k_2 = 0$ reduces to the law of Berman [62] which yields models with constant deceleration parameter. Solving (33), one obtained following different from of solution for the scale factor

$a = k_3 e^{k_4 t}$ For $k_2 = 0$ & $n = 0$

$a = k_5 (nt + k_4)^{\frac{1}{n}}$ For $k_2 = 0$ & $n > 0$.

where k_3, k_4, k_5, k_6 are constants of integration.

Case I

for $k_2 = 0$ & $n = 0$, the average scale factor is

$$a = k_3 e^{k_4 t} \tag{34}$$

From equations (32), (33) and (34) we get,

$$A = k_8 e^{k_4 t \left(2\sqrt{3}k_1 + 1\right)} \tag{35}$$

$$B = C = k_7 e^{k_4 (1 - \sqrt{3}k_1) t} \tag{36}$$

Where $k_7 = k_3^{\left(1-\sqrt{3}k_1\right)}$ and $k_8 = k_7^{\frac{2\sqrt{3}k_1 + 1}{1-\sqrt{3}k_1}}$

By using equations (35), (36) in equations (18) and (19) it gives average scale factor and spatial volume are as fallows

$$a(t) = k_9^{\frac{1}{3}} e^{k_4 t} \tag{37}$$

$$V = k_9 e^{3 k_4 t} \tag{38}$$

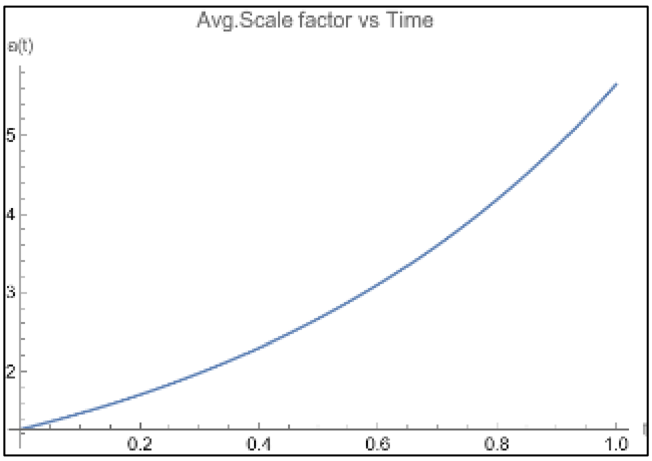

Figure 1 Average Scale factor $a(t)$ vs. time (t) for $k_9 = 2, k_4 = 1.5$

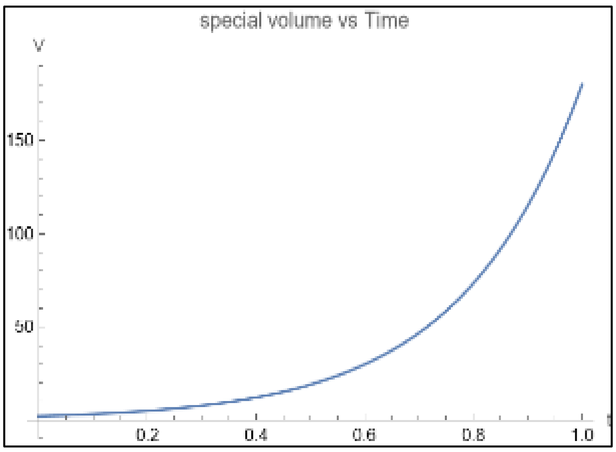

Figure 2 Spatial volume (V) Vs. time (t) for $k_9 = 2, k_4 = 1.5$

From Fig. (1) and Fig. (2), it is observed that at initial moment (time = 0), both the average scale factor and spatial volume have constant values. As time progresses, they increase continuously and eventually tend toward infinity.

The mean generalized Hubble's parameter H for this model defined by (20) takes the form

$$H = k_4 \tag{39}$$

The expansion scalar θ and shear scalar σ^2 are define by (22) and (23) takes the form

$$\theta = 3k_4 \tag{40}$$

$$\sigma^2 = 9(k_1 k_4)^2 \tag{41}$$

The average anisotropic parameter is,

$$A_m = 6k_1^2 \tag{42}$$

By using equation (35) and (36), in equation (3) takes the form

$$ds^2 = -dt^2 + k_8^2 e^{2k_4\left(2\sqrt{3}k_1+1\right)t} dx^2 + k_7^2 e^{2k_4\left(1-\sqrt{3}k_1\right)t} e^{2x} dy^2 + k_7^2 e^{2k_4\left(1-\sqrt{3}k_1\right)t} e^{-2x} dz^2 \tag{43}$$

Equation (29) has a solution of the form

$$\phi = \frac{k_{12}}{e^{3k_4 t}} \qquad \text{where, } k_{12} = -3k_{11}k_4 \tag{44}$$

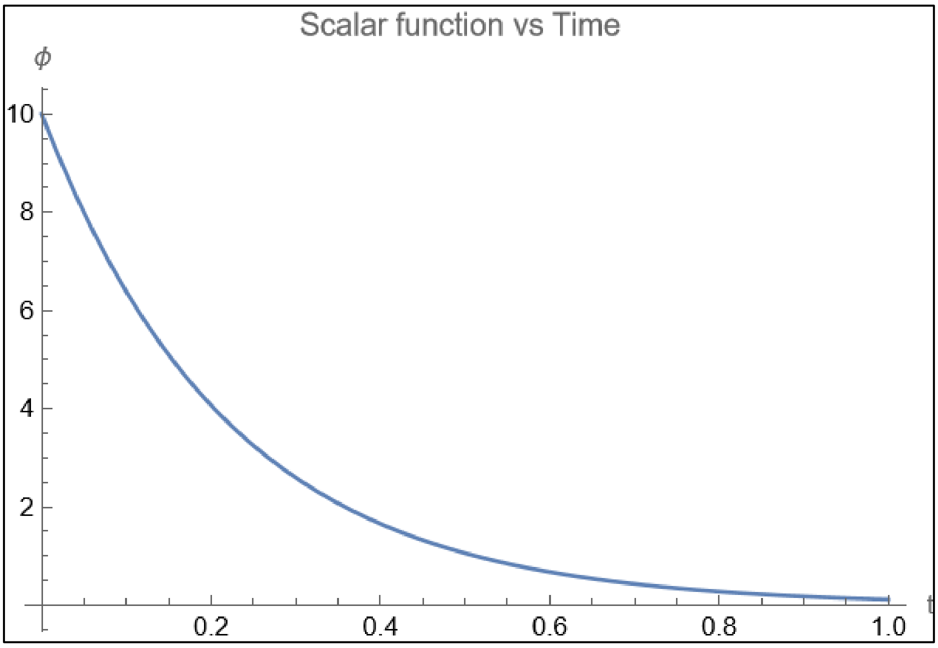

Figure 3 Scalar function (ϕ) vs. time (t) for $k_{12} = 10, k_4 = 1.5$

From Fig. (3) and equation (44), it is observed that initially, at $t = 0$, the scalar function has a constant value. As time increases, the scalar function gradually decreases and eventually tends toward zero.

Now, in order to find the value of density and pressure by using equation (35), (36) and equation (44) in equation (28) we get,

$$\rho = \frac{k_{12}}{8\pi e^{3k_4 t}}\left\{k_{13} - \frac{1}{k_8^2 e^{k_4\left(2\sqrt{3}k_1+1\right)t}} - \frac{D^2 e^{3k_4 t}}{k_{12}k_7^4 e^{4k_4\left(1-\sqrt{3}k_1\right)t}}\right\} \tag{45}$$

$$p = \frac{3k_{12}}{8\pi e^{3k_4 t}}\left\{k_{13} - \frac{1}{k_8^2 e^{k_4\left(2\sqrt{3}k_1+1\right)t}} - \frac{D^2 e^{3k_4 t}}{k_{12}k_7^4 e^{4k_4\left(1-\sqrt{3}k_1\right)t}}\right\} \tag{46}$$

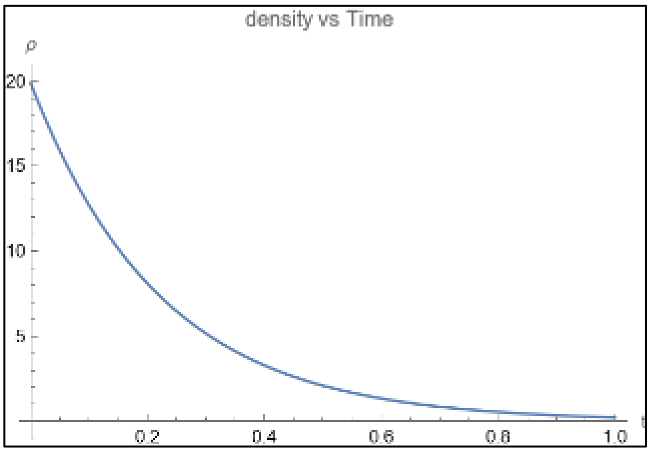

Figure 4 Density (ρ) vs. time (t) for $k_{12} = 10, k_{13} = 50, k_4 = 1.5, k_7 = k_8 = 5.5, k_1 = 1.1, D = 2$

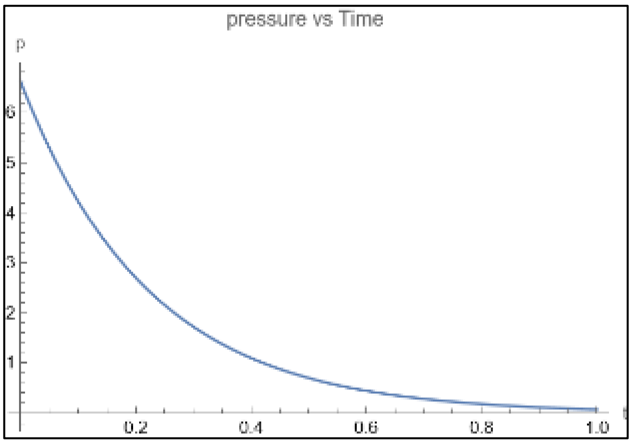

Figure 5 Pressure (p) vs. time (t) for $k_{12} = 10, k_{13} = 50, k_4 = 1.5, k_7 = k_8 = 5.5, k_1 = 1.1, D = 2$

From Fig. (4), Fig. (5), and equations (45) to (46), it is observed that as time progresses, both density and pressure decreases continuously. The model does not exhibit any singularity, as all physical quantities remain finite and constant at the initial moment.

4. Jerk Parameter

The jerk parameter is defined as (Visser [63])

$$j(t) = \frac{1}{H^3}\frac{\dddot{a}}{a} = q + 2q^2 - \frac{\dot{q}}{H} \tag{47}$$

Here, a is the average scale factor and H is the Hubble parameter and the dot denotes differentiation with respect to the time. Flat ΛCDM models have a constant jerk for solving equation (47) in case- I, the jerk parameter is found to be

$$j(t) = 1 \tag{48}$$

5. Statefinder diagnostic

Sahni et al. [64] Introduced a diagnostic proposal that makes use of the parameter pair {r, s}, the so-called 'statefinder.' The statefinder pair {r, s} is defined as

$$r = \frac{\dddot{a}}{aH^3}, \qquad s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} \tag{49}$$

For solving equation (49), the statefinder parameters are found to be

$$r = 1 \quad \text{and} \quad s = 0 \tag{50}$$

It can be observed from equation (50) the statefinder parameter is independent from time. And the values of r and s ultimately correspond to the derived model to ΛCDM.

5.1. Case II

For $k_2 = 0$ and $n > 0$ average scale factor is

$$a = k_5 (nt + k_4)^{\frac{1}{n}} \tag{51}$$

From equations (18), (32) and (51) we get,

$$A = k_{15} (nt + k_4)^{\frac{2\sqrt{3}k_1 + 1}{n}} \tag{52}$$

$$B = k_{14} (nt + k_4)^{\frac{1 - \sqrt{3}k_1}{n}} \tag{53}$$

By using equations (52), (53) in equation (18), (19) gives the values of average scale factor and spatial volume are as follow

$$a(t) = k_{16}^{\frac{1}{3}} (nt + k_4)^{\frac{1}{n}} \tag{54}$$

$$V = k_{16} (nt + k_4)^{\frac{3}{n}} \tag{55}$$

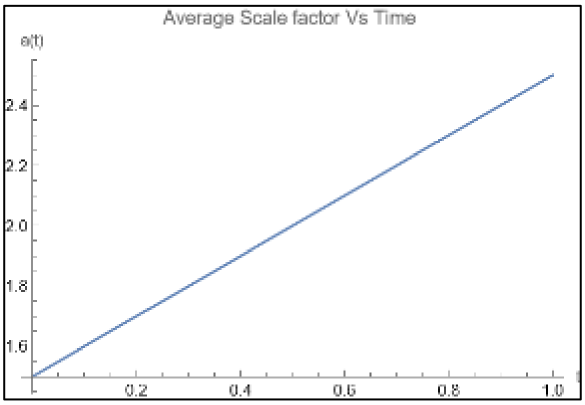

Figure 6 Average Scale factor $a(t)$ Vs. time (t) for $n = 1, k_{16} = 1, k_4 = 1.5$

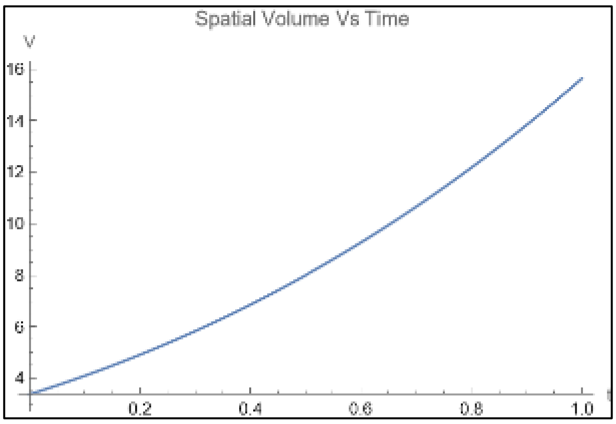

Figure 7 Spatial volume (V) Vs. time (t) for $n = 1, k_{16} = 1, k_4 = 1.5$

From Fig. (6), it is observed that the average scale factor increase with time and eventually tends toward infinity, indicating the continuous expansion of the universe. Similarly, from Fig. (7), it is seen that the spatial volume is initially constant and then increases with time, ultimately approaching infinity.

The mean generalized Hubble's parameter H for this model defined by (20) takes the form

$$H = \frac{1}{(nt + k_4)} \tag{56}$$

The expansion scalar θ and shear scalar σ^2 are defined by (22) and (23) takes the form

$$\theta = \frac{3}{(nt + k_4)} \tag{57}$$

$$\sigma^2 = \frac{9k_1^2}{(nt + k_4)^2} \tag{58}$$

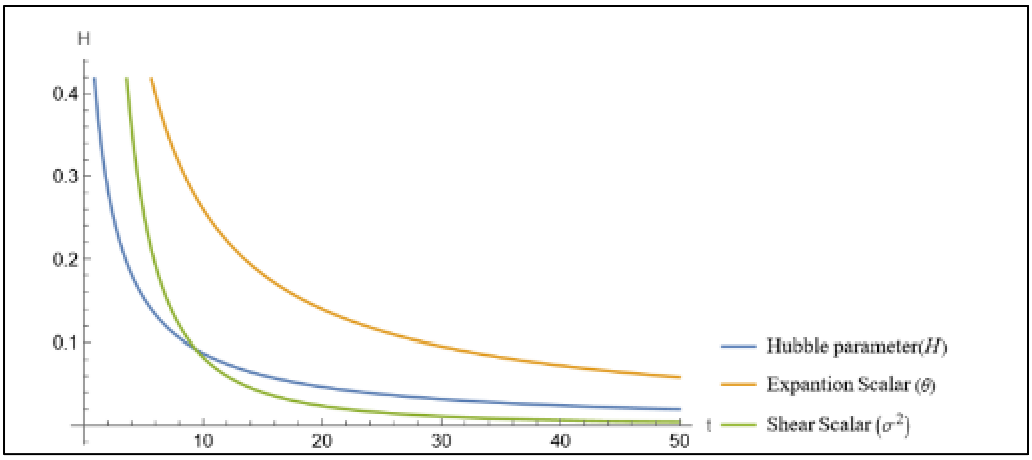

Figure 8 Hubble Parameter (H) ,Expansion Scalar (θ) ,Shear Scalar (σ) vs. time (t) for $n = 1, k_1 = 1.1, k_4 = 1.5$

From Fig.8 it is observed that Hubble parameter, Expansion scalar and Shear scalar also decrease gradually as time increases.

The average anisotropic parameter is

$$A_m = 6k_1^2 \tag{59}$$

By using equations (52) and (53), equation (3) takes the form

$$ds^2 = -dt^2 + \left(k_{15}(nt + k_4)^{\frac{2\sqrt{3}k_1 + 1}{n}} \right)^2 dx^2 + \left(k_{14}(nt + k_4)^{\frac{1 - \sqrt{3}k_1}{n}} \right)^2 e^{2x} dy^2 + \left(k_{14}(nt + k_4)^{\frac{1 - \sqrt{3}k_1}{n}} \right)^2 e^{-2x} dz^2 \tag{60}$$

Equation (29) has a solution is of the form

$$\phi = \frac{k_{19}}{(nt + k_4)^{3 + \sqrt{3}k_1}} \tag{61}$$

To find the value of density and pressure by using equation (52), (53) and equation (61) in equation (28) we get

$$\rho = \frac{k_{19}}{8\pi(nt+k_4)^{3+\sqrt{3}k_1}}\left[\frac{2\left(2\sqrt{3}k_1+1\right)\left(1-\sqrt{3}k_1\right)}{(nt+k_4)^2}+\frac{\left(1-\sqrt{3}k_1\right)^2}{(nt+k_4)^2}-\frac{1}{k_{15}^2(nt+k_4)^{\frac{4\sqrt{3}k_1+2}{n}}}-\frac{D^2(nt+k_4)^{3+\sqrt{3}k_1}}{k_{19}k_{14}^4(nt+k_4)^{\frac{4\left(1-\sqrt{3}k_1\right)}{n}}}\right] \tag{62}$$

$$p = \frac{3k_{19}}{8\pi(nt+k_4)^{3+\sqrt{3}k_1}}\left[\frac{2\left(2\sqrt{3}k_1+1\right)\left(1-\sqrt{3}k_1\right)}{(nt+k_4)^2}+\frac{\left(1-\sqrt{3}k_1\right)^2}{(nt+k_4)^2}-\frac{1}{k_{15}^2(nt+k_4)^{\frac{4\sqrt{3}k_1+2}{n}}}-\frac{D^2(nt+k_4)^{3+\sqrt{3}k_1}}{k_{19}k_{14}^4(nt+k_4)^{\frac{4\left(1-\sqrt{3}k_1\right)}{n}}}\right] \tag{63}$$

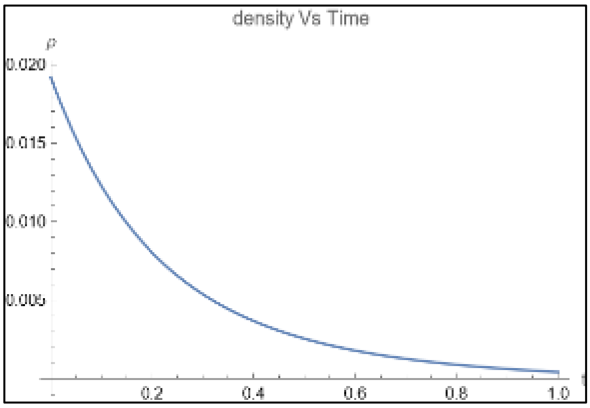

Figure 9 Density (ρ) vs. time (t) for $k_1 = 1.1, k_4 = 1.5, k_{14} = k_{15} = 1 = n,$ $k_{19} = -1, D = 0.01$

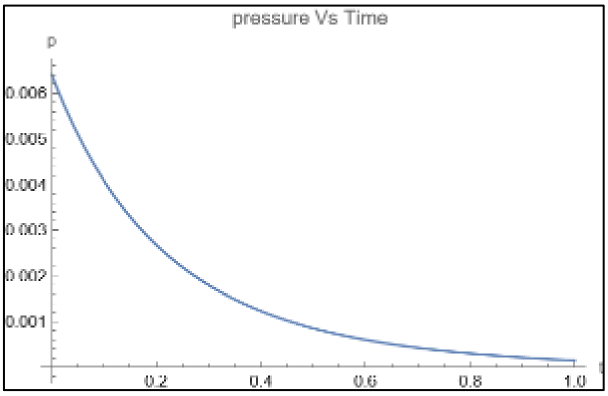

Figure 10 Pressure (p) vs. time (t) for $k_1 = 1.1, k_4 = 1.5, k_{14} = k_{15} = 1 = n,$ $k_{19} = -1, D = 0.01$

From Fig. 9 and Fig. 10, it is observed that the density and pressure decreasing with respective time t and approaches to zero after some finite time for choose constant.

6. Deceleration parameter

The deceleration parameter is define as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{64}$$

Where a refers to scale factor and dots denote the derivative with respect to time. The sign of deceleration parameter indicate whether the model is accelerating or decelerating. The positive sign indicates a decelerating universe. Also, negative sign indicates an accelerating expansion of the universe and when $q = 0$ there exits marginal inflection.

Solving equation (64) gives,

$$q = -1 + n, \qquad n \neq 0$$

$$q = -1, \qquad n = 0$$

In this case, it is observed that for $n < 1$ the deceleration parameter represents accelerated universe whereas $n > 1$ will represent decelerated universe, in this way, $q = 0$ for $n = 1$ represents expanding universe with constant velocity.

7. Jerk Parameter

The jerk parameter is define as

$$j(t)=\frac{\dddot{a}}{aH^3}=q+2q^2-\frac{\dot{q}}{H} \tag{65}$$

Here a is scale factor, H is Hubble parameter and dot denotes differentiation with respective time. The jerk parameter is positive throughout the evolution of the universe, and it attains a constant value at late time. Hence the expansion in the model from decelerating phase to attain accelerating phase is smooth one. For solving equation (65) the jerk parameter gives

$$j(t)=1-3n+2n^2 \qquad n\neq 0 \tag{66}$$

$$j(t)=1 \qquad n=0 \tag{67}$$

In this case it is observed that, the jerk is independent from time and the value of jerk parameter is unity. Hence model is flat ΛCDM.

8. Statefinder parameter

The Statefinder parameter, introduced by Sahani et al. [64] are indeed an important tool in modern cosmology to distinguish between different models of dark energy. These parameters are constructed by combining the Hubble parameter H and deceleration parameter q, providing a more detailed description of the evolution of the universe. The two new parameters, r and s are define as

$$r=\frac{\dddot{a}}{aH^3} \qquad \text{and} \qquad s=\frac{r-1}{3\left(q-\frac{1}{2}\right)} \tag{68}$$

In this case, the statefinder parameters are found to be

$$r=1-3n+2n^2 \qquad \text{and} \qquad s=\frac{3n-2n^2}{3\left(n-\frac{3}{2}\right)} \qquad \text{for } n\neq 0 \tag{69}$$

$$r=1 \quad \text{and} \quad s=0 \qquad \text{for } n=0 \tag{70}$$

It can be observed that from equation (69) and (70) the statefinder parameters are independent from time, the values of r and s ultimately correspond to the derived model to ΛCDM .

9. The Thermodynamic Quantities of the Universe: Entropy, Enthalpy, Gibbs free energy and Helmholtz's free Energy

According to the second law of thermodynamics, the total internal energy of any system (including the universe) cannot be completely converted into useful work. Only a part of this energy, known as available energy, can be utilized to perform useful work, while the remaining portion, which cannot be converted into work, is referred to as unavailable energy. Entropy is a measure of this unavailable energy. More precisely, entropy can be defined as the amount of unavailable energy per unit temperature. In other words, it represents the degree of disorder or randomness present within a system (or the universe). The entropy will be denoted by S .

For the universe the enthalpy or heat content, represented by H, which you can think of as the energy already in the system. Gibbs free energy is the energy available in the universe to do work. That means energy that is not dissipated through heat or expansion of the universe and the part of the internal energy which is used in useful work, named Helmholtz's free energy F.

If we use ρV as a definition for the internal energy U of the universe, the Helmholtz free energy F, Enthalpy H and Gibbs free energy G can be defined as follow

The entropy S is reads as

$$S = \frac{A}{4G} \tag{71}$$

where, $A = 4\pi R_h^2, \quad R_h = \frac{1}{H}$ indicates as Hubble horizon and H is the Hubble parameter.

Considering the universe's internal energy $U = \rho V$ and temperature T

$$T = \frac{H(meanhubbleParameter)}{2\pi} \tag{72}$$

The enthalpy H, Helmholtz energy F, Gibbs energy G are define as follows

$$H = U + pV = (\rho + p)V \tag{73}$$

$$F = U - TS = \rho V - TS \tag{74}$$

$$G = H - TS \tag{75}$$

Where T is the Temperature

By using equation (39) in equation (71) and (72) the entropy and temperature are takes the form for case I.

$$S = \frac{\pi}{k_4^2 G} \tag{76}$$

$$T = \frac{k_4}{2\pi} \tag{77}$$

The values for enthalpy H, Helmholtz energy F and Gibbs energy G are as follows for Case I

$$H = \frac{k_9}{6\pi}\left\{k_{12}k_{13} - \frac{k_{12}}{k_8^2 e^{2k_4(2\sqrt{3}k_1+1)t}} - \frac{D^2 e^{3k_4 t}}{k_{12}k_7^4 e^{4k_4(1-\sqrt{3}k_1)t}}\right\} \tag{78}$$

$$F = \frac{k_9}{8\pi}\left\{k_{12}k_{13} - \frac{k_{12}}{k_8^2 e^{k_4(2\sqrt{3}k_1+1)t}} - \frac{D^2 e^{3k_4 t}}{k_7^4 e^{4k_4(1-\sqrt{3}k_1)t}}\right\} - \frac{1}{2k_4 G} \tag{79}$$

$$G = \frac{k_9}{6\pi}\left\{k_{12}k_{13} - \frac{k_{12}}{k_8^2 e^{2k_4(2\sqrt{3}k_1+1)t}} - \frac{D^2 e^{3k_4 t}}{k_{12}k_7^4 e^{4k_4(1-\sqrt{3}k_1)t}}\right\} - \frac{1}{2k_4 G} \tag{80}$$

By using equation (76) in equation (71) and (72) the entropy and temperature are takes the form for Case II

$$s=\frac{\pi(nt+k_4)^2}{G} \tag{81}$$

$$T=\frac{1}{2\pi(nt+k_4)} \tag{82}$$

The values for enthalpy H , Helmholtz energy F , Gibbs energy G are as follows

$$H=\frac{k_{19}k_{16}(nt+k_4)^{\frac{3}{n}}}{6\pi(nt+k_4)^{3+\sqrt{3}k_1}}\left[\frac{2(2\sqrt{3}k_1+1)(1-\sqrt{3}k_1)}{(nt+k_4)^2}+\frac{(1-\sqrt{3}k_1)^2}{(nt+k_4)^2}-\frac{1}{k_{15}^2(nt+k_4)^{\frac{4\sqrt{3}k_1+2}{n}}}-\frac{D^2(nt+k_4)^{3+\sqrt{3}k_1}}{k_{19}k_{14}^4(nt+k_4)^{\frac{4(1-\sqrt{3}k_1)}{n}}}\right] \tag{83}$$

$$F=\frac{k_{19}k_{16}(nt+k_4)^{\frac{3}{n}}}{8\pi(nt+k_4)^{3+\sqrt{3}k_1}}\left[\frac{2(2\sqrt{3}k_1+1)(1-\sqrt{3}k_1)}{(nt+k_4)^2}+\frac{(1-\sqrt{3}k_1)^2}{(nt+k_4)^2}-\frac{1}{k_{15}^2(nt+k_4)^{\frac{4\sqrt{3}k_1+2}{n}}}-\frac{D^2(nt+k_4)^{3+\sqrt{3}k_1}}{k_{19}k_{14}^4(nt+k_4)^{\frac{4(1-\sqrt{3}k_1)}{n}}}\right]-\frac{(nt+k_4)}{2G} \tag{84}$$

$$G=\frac{k_{19}k_{16}(nt+k_4)^{\frac{3}{n}}}{6\pi(nt+k_4)^{3+\sqrt{3}k_1}}\left[\frac{2(2\sqrt{3}k_1+1)(1-\sqrt{3}k_1)}{(nt+k_4)^2}+\frac{(1-\sqrt{3}k_1)^2}{(nt+k_4)^2}-\frac{1}{k_{15}^2(nt+k_4)^{\frac{4\sqrt{3}k_1+2}{n}}}-\frac{D^2(nt+k_4)^{3+\sqrt{3}k_1}}{k_{19}k_{14}^4(nt+k_4)^{\frac{4(1-\sqrt{3}k_1)}{n}}}\right]-\frac{(nt+k_4)}{2G} \tag{85}$$

10. Conclusions

In this paper, we have explored a Bianchi type VI_0 cosmological model consisting of a perfect fluid coupled with an electromagnetic field within the framework of self-creation theory of gravitation. We analyzed two distinct solution cases. To obtain determinate solutions, we employed the condition that the shear scalar is proportional to the scalar expansion, and we also considered a generalized linearly varying deceleration parameter (as proposed by Akarsu and Dereli) along with the equation of radiation. In both cases, the average scale factor $a(t)$ and spatial volume V are constant at the initial epoch and then grow exponentially with time, indicating the continuous expansion of the universe. The anisotropic parameter remains constant in both cases. In case I, the Hubble parameter (H), Expansion scalar (θ), and Shear scalar (σ) are constant, whereas in Case II, these parameters gradually decrease over time and eventually approach constant values. In both cases, the scalar function decreases as time progresses. Furthermore, for both models, the density and pressure decrease with time and approach zero after a finite duration for the chosen constant. Notably, the model exhibits no singularity since all physical quantities are finite and constant at the initial time.

In case I and II it is observed that, the jerk parameter is independent from time and the value of jerk parameter is unity. Hence model is flat ΛCDM It can be observed that from Case I and II the statefinder parameters are independent from time, the values of (r,s) ultimately correspond to the derived model to ΛCDM . In Both cases, several thermodynamic properties are examined, including entropy (S), temperature (T), Enthalpy (H), Helmholtz energy (F) and Gibbs energy (G) in case II it is observed that all the properties of thermodynamic are increasing functions with time. Also, both cases $\lim_{t\to\infty}\left(\frac{\sigma}{\theta}\right)\neq 0$ the model does not approach isotropy for large value of t.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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