



(RESEARCH ARTICLE)



A fuzzy logic controller for nonlinear inverted pendulum systems: Design, simulation, and performance evaluation

Chieu Hanh Vu *, Duc Hong Nguyen and Trinh Hieu Tran

Department of Electrical and Electronics Engineering, Faculty of Electrical & Electronics Engineering, Ly Tu Trong College, Ho Chi Minh City, Viet Nam.

International Journal of Science and Research Archive, 2025, 17(02), 741–753

Publication history: Received on 03 October 2025; revised on 13 November 2025; accepted on 15 November 2025

Article DOI: <https://doi.org/10.30574/ijrsra.2025.17.2.3004>

Abstract

The inverted pendulum on a cart is a classical benchmark used to evaluate control strategies for nonlinear and underactuated systems. Its inherent instability and strong coupling between the pendulum's rotation and the cart's translation make it a challenging system to stabilize. This paper proposes a fuzzy logic-based controller (FLC) designed to stabilize the pendulum in its upright position while maintaining the cart near its equilibrium point. Unlike traditional linear controllers that require precise modeling or system linearization, the FLC uses linguistic rules and triangular membership functions to manage nonlinearities and uncertainties. A nonlinear mathematical model of the pendulum-cart system is developed and implemented in MATLAB/Simulink, where the fuzzy controller computes the control force based on real-time feedback of angular and translational states. Simulation results demonstrate that the proposed FLC achieves fast stabilization with a settling time of less than 5 seconds, minimal overshoot, and smooth transient performance. The control input remains bounded and energy-efficient, and the system maintains stability under disturbances and parameter variations. Overall, the results confirm that the fuzzy logic controller provides a robust, adaptive, and interpretable solution for nonlinear dynamic systems, outperforming traditional PID and model-based controllers in terms of response speed, stability, and robustness.

Keyword: Inverted Pendulum; Fuzzy Logic Control; Nonlinear Dynamics; System Stability; Intelligent Control

1. Introduction

The Inverted Pendulum and Cart System is a well-known benchmark in control engineering, widely used to study the stabilization of nonlinear and underactuated dynamic systems. Structurally, it consists of a pendulum attached to a movable cart, where the pendulum's center of mass lies above the pivot point, resulting in an inherently unstable equilibrium configuration. The cart can move horizontally under the influence of an external control force, while the pendulum is free to rotate about the pivot. Unlike fully actuated systems, the inverted pendulum exhibits strong nonlinear coupling between translational and rotational motions. The movement of the cart directly affects the pendulum's angular displacement, and vice versa, creating a challenging dynamic interaction. This coupling, combined with gravitational torque and limited actuation, makes the system highly sensitive to external disturbances, parameter variations, and modeling inaccuracies.

Fuzzy logic control (FLC) has been widely adopted as an effective approach for nonlinear and uncertain systems, particularly where accurate mathematical models are difficult to obtain. Unlike conventional controllers such as PID or LQR, which typically require linearization around an operating point and precise system identification, FLC relies on fuzzy sets and rule-based inference to approximate nonlinear dynamics and incorporate expert knowledge directly into the control law (Tang & Ahmad, 2024). This property allows FLC to maintain stability and robustness under parameter

* Corresponding author: Chieu Hanh Vu

variations and external disturbances, where classical linear controllers often fail. Another important advantage of FLC lies in its interpretability and design flexibility. Unlike black-box models such as neural networks, fuzzy rules can be easily adjusted or expanded by engineers, facilitating transparency and practical deployment (Tang & Ahmad, 2024). Furthermore, the performance of FLC can be significantly enhanced when combined with optimization techniques. Melese et al. (2025) recently demonstrated that an optimized fuzzy–PID controller, tuned by evolutionary algorithms, provided improved transient response and tracking accuracy in object-following applications. Collectively, these results confirm that FLC offers a robust and adaptable paradigm that overcomes several inherent limitations of traditional linear controllers, making it a strong candidate for controlling complex nonlinear systems such as the inverted wedge mechanism.

In recent years, the use of fuzzy logic controllers (FLCs) for handling nonlinear systems has attracted considerable research interest, owing to their ability to manage uncertainties, nonlinearities, and model inaccuracies in a systematic yet interpretable way. For instance, Su et al. (2021) developed an event-triggered output feedback fuzzy controller for discrete-time Takagi–Sugeno systems, demonstrating stability under measurement and modeling uncertainties. In the domain of fractional-order systems, Zhang et al. (2025) proposed an adaptive fuzzy command filtered control strategy for uncertain nonlinear fractional systems, achieving robust tracking in the presence of disturbances. Meanwhile, Al-Hadithi et al. (2025) introduced a control framework based on augmented incremental state-space modeling combined with Takagi–Sugeno fuzzy models, enabling enhanced adaptability and precision in nonlinear control tasks. In another line of work, Castillo et al. (2021) proposed a fuzzy-fractal control method that fuses fractal geometry and fuzzy rules to better capture the complexity of nonlinear dynamics, showing promise in effectively moderating oscillatory behavior. Also, Bao et al. (2022) designed an adaptive fuzzy control and optimization architecture specifically for nonlinear systems subject to uncertainties and disturbances, providing a constructive way to tune rule bases and membership functions in real time. Roose and Yahya (2017) developed a classical fuzzy logic controller for the pendulum–cart system and demonstrated its effectiveness in maintaining both angular and positional stability using MATLAB/Simulink simulations. Their results highlighted the ability of fuzzy control to handle the nonlinear coupling between translational and rotational dynamics without requiring an exact mathematical model.

Building upon this, Mahmoodabadi et al. (2020) proposed an optimal robust adaptive fuzzy controller capable of compensating for parameter variations and external disturbances. Their approach improved transient response and robustness, confirming that fuzzy-based adaptation enhances control performance for nonlinear dynamic systems. More recently, Nguyen et al. (2023) introduced a Takagi–Sugeno fuzzy control model with disturbance rejection and input–output constraints, further extending the robustness of fuzzy control in complex environments. These studies collectively reinforce the suitability of fuzzy logic control as an effective and interpretable technique for stabilizing nonlinear and underactuated systems such as the inverted pendulum.

Despite its simplicity in structure, designing an effective fuzzy logic controller for the inverted pendulum and cart system remains a highly challenging task. The system’s inherent nonlinearity, underactuated nature, and sensitivity to parameter variations require a carefully designed fuzzy inference system with well-tuned membership functions and optimized control rules. This research aims to develop a fuzzy logic–based control strategy specifically tailored for the inverted pendulum and cart system to ensure fast stabilization, smooth transient performance, and robustness against disturbances. The proposed approach integrates fuzzy reasoning with systematic normalization and scaling to enhance control precision and stability. The effectiveness of the controller is verified through MATLAB/Simulink simulations, and its performance is evaluated under various initial conditions and compared with conventional PID and linear control methods to highlight its superior capability in handling nonlinear dynamics.

The main contributions of this study can be summarized as follows:

- Designing a fuzzy logic controller to stabilize the inverted pendulum and cart system by effectively managing the nonlinear coupling between angular and translational motions.
- Developing and tuning fuzzy membership functions and scaling parameters to achieve optimal transient response and minimal settling time.

2. Relevant theory

2.1. The Ball and Beam System

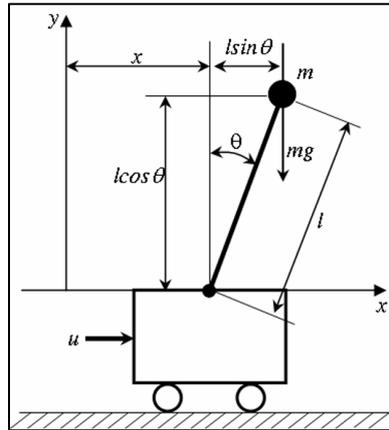


Figure 1 Inverted Pendulum and Cart System

2.1.1. Mathematical Model

The inverted pendulum system in Figure 1 is a classical nonlinear and underactuated mechanical system that exhibits coupled translational and rotational dynamics. The system consists of a cart of mass M that moves horizontally along a track, and a pendulum of mass m and length l pivoted on the cart, which can rotate in the vertical plane. The control input u is the horizontal force applied to the cart, generating translational motion and indirectly influencing the pendulum's angular behavior. Due to gravity, the pendulum's upright position represents an inherently unstable equilibrium.

The inverted pendulum on a cart is governed by the coupled dynamics between the horizontal motion of the cart and the angular motion of the pendulum. The system can be modeled using either Newton-Euler or Lagrangian mechanics to capture the nonlinear relationship between the applied force, gravitational effects, and the interaction between translational and rotational components.

Let x denote the horizontal displacement of the cart, θ the angular deviation of the pendulum from the upright position, M and m the masses of the cart and the pendulum respectively, l the pendulum length, and I the moment of inertia of the pendulum about its pivot. The control input u represents the horizontal force acting on the cart. Viscous damping coefficients b_1 and b_2 are included to account for frictional effects in the pendulum pivot and the cart's motion, respectively.

Applying Newton-Euler equations to both the translational and rotational components yields two coupled second-order differential equations describing the system dynamics:

$$\begin{aligned} (M + m)\ddot{x} + b_2\dot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta &= u \\ (I + ml^2)\ddot{\theta} + b_1\dot{\theta} + mgl\sin\theta &= -ml\ddot{x}\cos\theta \end{aligned}$$

Solving these equations simultaneously for \ddot{x} and $\ddot{\theta}$, the nonlinear model of the inverted pendulum system can be expressed as:

$$\begin{aligned} \ddot{x} &= \frac{(u + ml\dot{\theta}^2\sin\theta - b_2\dot{x})l - (mgl\sin\theta - b_1\dot{\theta})\cos\theta}{(M + m)l - ml\cos^2\theta} \\ \ddot{\theta} &= \frac{(mgl\sin\theta - b_1\dot{\theta})(M + m) - (u + ml\dot{\theta}^2\sin\theta - b_2\dot{x})ml\cos\theta}{(M + m)ml^2 - m^2l^2\cos^2\theta} \end{aligned}$$

These equations describe the nonlinear coupled dynamics between the cart's horizontal acceleration and the pendulum's angular acceleration. The nonlinearities arise from trigonometric terms ($\sin \theta$, $\cos \theta$), centrifugal terms $ml\dot{\theta}^2 \sin \theta$, and the product term $ml\ddot{x} \cos \theta$, which create strong interdependence between the two motions.

Due to the gravitational torque acting on the pendulum, the upright position ($\theta = 0$) represents an unstable equilibrium point. Consequently, any small disturbance can cause the pendulum to deviate significantly, necessitating an active control strategy to stabilize the system. The inverted pendulum thus serves as a canonical example for evaluating advanced nonlinear control methods—such as fuzzy logic, adaptive, and reinforcement learning-based approaches—capable of handling complex nonlinear dynamics and real-time balancing control.

2.1.2. Dynamic System Behavior

Nonlinear Dynamics

The nonlinear behavior of the inverted pendulum system primarily arises from the strong coupling between the translational motion of the cart and the rotational motion of the pendulum around its pivot. The governing equations of motion include multiple nonlinear components, such as trigonometric terms ($\sin \theta$, $\cos \theta$), velocity-coupling terms ($\dot{\theta}^2 \sin \theta$), and mixed acceleration terms involving both translational and angular accelerations ($\ddot{x} \cos \theta$). These nonlinearities originate from the pendulum's gravitational torque and the inertial coupling between the pendulum's rotation and the cart's horizontal motion.

Small variations in the pendulum's angular displacement θ or the cart's position x can lead to significant changes in the overall system response because of this dynamic interdependence. The trigonometric relationship between the gravitational restoring torque and the pendulum's angular motion introduces both stabilizing and destabilizing effects depending on the pendulum's position relative to the vertical. As a result, the system exhibits strongly coupled nonlinear dynamics, where translational and rotational motions influence each other through both kinematic and inertial interactions.

This nonlinear interaction makes it impossible to accurately describe the system using linear approximations for large angular deviations or high-speed cart movements. The pendulum's tendency to fall under gravity amplifies the instability whenever the control force is insufficient to compensate for nonlinear coupling effects. Therefore, the inverted pendulum system serves as a canonical example of a nonlinear underactuated system, requiring the implementation of advanced nonlinear or intelligent control strategies—such as fuzzy logic, adaptive control, or reinforcement learning—to effectively handle its nonlinearities and achieve stable balance and motion control.

2.1.3. Instability

The inverted pendulum system is inherently unstable due to the position of its center of mass being above the pivot point. In the upright equilibrium configuration, any small deviation in the pendulum's angular displacement θ from the vertical causes the gravitational torque to act in a direction that increases the deviation rather than restoring it. This positive feedback behavior makes the upright position dynamically unstable — even a minor disturbance or modeling inaccuracy can cause the pendulum to fall if not counteracted by active control.

Furthermore, the translational motion of the cart adds an additional layer of instability. When the pendulum tilts, the control force applied to the cart must simultaneously move the base to counterbalance the pendulum's fall while preventing the cart from drifting excessively. However, the nonlinear coupling between the pendulum's angular dynamics and the cart's translational dynamics means that rapid corrective movements can introduce oscillations or overshoot, leading to further instability if not properly managed.

2.2. Fuzzy Logic Controller

The pole cart, being a highly nonlinear and underactuated mechanical structure, represents an ideal application for FLC. The strong coupling between the translational motion of the sliding mass and the rotational dynamics of the wedge introduces nonlinear interactions that challenge classical linear control approaches. FLC, on the other hand, can handle such nonlinear relationships by mapping expert knowledge into a set of fuzzy rules that relate measurable variables—such as the wedge's angle, angular velocity, and the cart's position—to control actions in a continuous and adaptive manner.

2.3. Define Input and Output Variables

In the fuzzy logic control (FLC) design for the inverted wedge system, the selection of appropriate input and output variables plays a crucial role in achieving effective stabilization and precise motion control.

2.3.1. Controlled plant

The controlled system is the cart–pendulum mechanism. A horizontal control force u is applied to the cart to regulate the pendulum and the cart motion.

- **Plant input:** control force u [N].
- **Plant outputs:** pendulum angle θ [rad] and cart position x [m]. In implementation we also use their time derivatives $\dot{\theta}$ and \dot{x} (measured or estimated) as feedback signals.

2.3.2. Fuzzy controller

The FLC generates the control action u based on the deviation of the system from its upright, centered equilibrium.

- **FLC inputs:** normalized (or scaled) error-like signals of the four feedback variables,

$$\bar{\theta} = K_1 \theta, \bar{\dot{\theta}} = K_2 \dot{\theta}, \bar{x} = K_3 x, \bar{\dot{x}} = K_4 \dot{x},$$

which capture the pendulum's angular state and the cart's translational state.

- **FLC output:** a normalized control command \bar{u} that is converted to the physical actuation by a scaling gain K_u ,

$$u = K_u \bar{u}.$$

Normalize Input and Output Variables

To standardize the variables for fuzzy processing, all input and output signals are normalized to the range of $[-1,1]$ for computation within the fuzzy inference system. The normalization gains are defined as follows:

- Pendulum angle (θ):
- $-0.3 \leq \theta \leq 0.3$ rad, with $K_1 = \frac{1}{0.3}$ for normalization.
- Angular velocity ($\dot{\theta}$):
- $-1 \leq \dot{\theta} \leq 1$ rad/s, with $K_2 = 1$ for normalization.
- Cart position (x):
- $-3 \leq x \leq 3$ m, with $K_3 = \frac{1}{3}$ for normalization.
- Cart velocity (\dot{x}):
- $-3 \leq \dot{x} \leq 3$ m/s, with $K_4 = \frac{1}{3}$ for normalization.
- Control force (u):
- $-40 \leq u \leq 40$ N, with $K_u = 40$ for normalization.

2.4. Define Fuzzy Sets and Linguistic Terms

The input and output variables of the fuzzy logic controller for the inverted pendulum on a cart system are described using fuzzy sets with corresponding linguistic terms, as illustrated in Fig. X. Each input variable is normalized to the range $[-1,1]$, and the associated fuzzy sets are defined to represent the qualitative states of the system's motion.

- Pendulum angle (θ): Negative (NE), Zero (ZE), and Positive (PO).

These linguistic terms describe the deviation of the pendulum from the vertical position.

- Angular velocity ($\dot{\theta}$): NE, ZE, and PO.

This variable captures the direction and rate of the pendulum’s angular motion.

- Cart position (x): NE, ZE, and PO.

These terms represent the lateral displacement of the cart from the reference position.

- Cart velocity (\dot{x}): NE, ZE, and PO.

The linguistic states describe the translational motion of the cart along the horizontal axis.

- Control force (u): Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (ZE), Positive Small (PS), Positive Medium (PM), and Positive Big (PB).

This output variable determines the magnitude and direction of the actuation force applied to the cart to maintain the pendulum’s upright stability and the cart’s position balance.

Each fuzzy set is represented by a triangular membership function (as shown in Figure 2), which quantifies the degree of membership for each linguistic term. The simple triangular structure ensures computational efficiency and smooth transition between adjacent fuzzy regions, enabling real-time control of the nonlinear inverted pendulum–cart system.

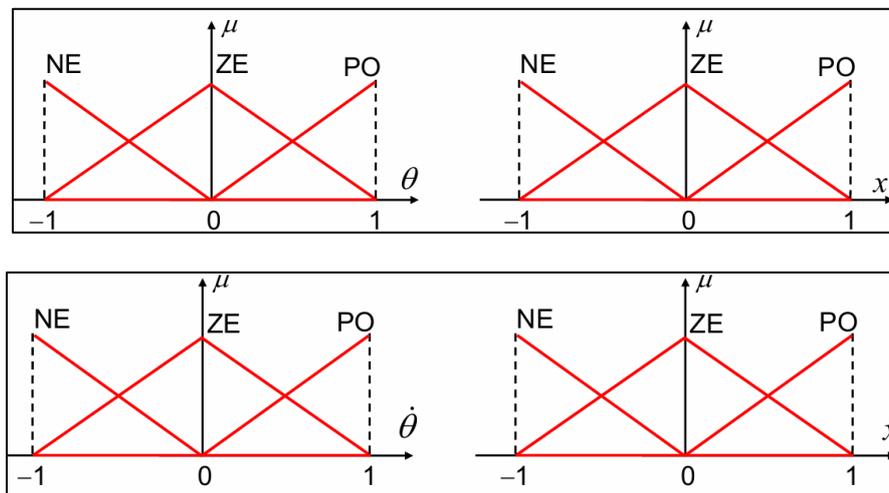


Figure 2 The Fuzzy Set of Controller

2.5. Formulate Fuzzy Rules

Using domain knowledge and system behavior, a set of fuzzy rules is defined to relate the input variables ($x, \dot{x}, \theta, \dot{\theta}$) to the output variable (u) and illustrated in Figure 3

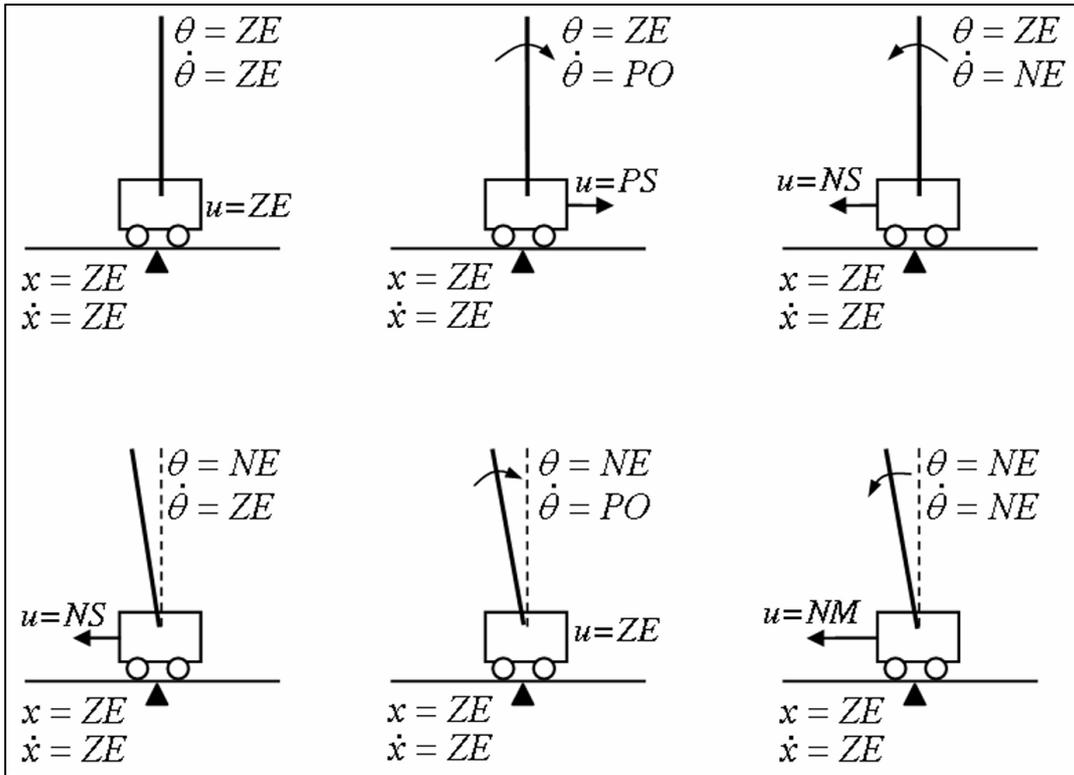


Figure 3 Illustration of fuzzy rules

2.6. Create a Rule Table

The fuzzy rules are organized into a rule table in table 1, ensuring symmetry and adjacency for consistent decision-making

Table 1 Full Fuzzy rule table

U			Theta									
			NE			ZE			PO			
			Theta dot									
			NE	ZE	PO	NE	ZE	PO	NE	ZE	PO	
X	NE	X dot	NE	NB	NB	NM	NB	NM	NS	NM	NS	ZE
			ZE	NB	NM	NS	NM	NS	ZE	NS	ZE	PS
			PO	NM	NS	ZE	NS	ZE	PS	ZE	PS	PM
	ZE		NE	NB	NM	NS	NM	NS	ZE	NS	ZE	PS
			ZE	NM	NS	ZE	NS	ZE	PS	ZE	PS	PM
			PO	NS	ZE	PS	ZE	PS	PM	PS	PM	PB
	PO		NE	NM	NS	ZE	NS	ZE	PS	ZE	PS	PM
			ZE	NS	ZE	PS	ZE	PS	PM	PS	PM	PB
			PO	ZE	PS	PM	PS	PM	PB	PM	PB	PB

2.7. Select Inference and Defuzzification Methods

2.7.1. Inference Method

The MAX-PROD inference method is adopted, in which the firing strength of each fuzzy rule is computed as the product of the membership degrees of all input variables. Among all activated rules, the maximum value is selected to determine the aggregated output strength. This approach provides smooth control action and ensures numerical stability for real-time pendulum balancing.

2.7.2. Defuzzification Method:

The weighted average (centroid) method is employed to compute the crisp output control force u . This method provides a balanced and continuous control signal, allowing the fuzzy controller to generate an appropriate horizontal actuation force that stabilizes the pendulum in the upright position while maintaining the cart near its reference location.

3. Proposed Method for Simulation using MATLAB/Simulink

A comprehensive simulation study is performed in MATLAB/Simulink to assess the effectiveness of the proposed fuzzy logic-based control approach in stabilizing and controlling the inverted pendulum on a cart. The simulation environment is designed to accurately reproduce the nonlinear dynamic behavior of the pendulum–cart mechanism, implement the fuzzy logic control algorithm, and evaluate the system’s performance under various dynamic and disturbance conditions.

3.1. System Modeling in Simulink

The inverted pendulum system is modeled in MATLAB/Simulink using the nonlinear differential equations derived in the previous section (Figure 4). This model reflects the inherent coupling between the pendulum’s angular motion and the cart’s translational motion and forms the foundation for implementing and testing advanced control strategies such as fuzzy logic control. The main components of the Simulink model can be described as follows:

3.2. Pendulum–Cart Dynamics

The nonlinear dynamics of the system are governed by two coupled differential equations that describe the interaction between the pendulum and the cart. These equations capture the effects of gravity, inertia, and damping, and establish the relationship between the control force u , the pendulum’s angular displacement θ , and the cart’s horizontal position x .

3.3. Control Force Generation

The horizontal control force u applied to the cart serves as the main actuation signal influencing both the angular acceleration of the pendulum and the linear motion of the cart. The fuzzy logic controller computes this control force based on the deviation of the pendulum’s angle and the cart’s position from their equilibrium values, enabling effective stabilization and precise trajectory tracking.

3.3.1. Nonlinearities and External Disturbances:

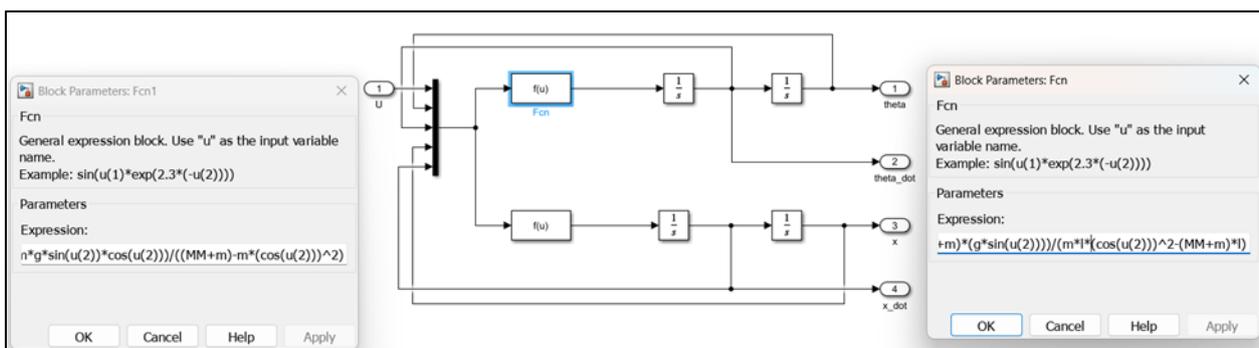


Figure 4 Simulink model of Inverted Pendulum and Cart system

The simulation explicitly accounts for the nonlinear characteristics of the pendulum–cart system, including trigonometric dependencies ($\sin \theta$, $\cos \theta$), dynamic coupling between angular and translational motions, and frictional effects. To evaluate the robustness of the proposed controller, the model also introduces external disturbances such as random perturbations, actuator time delays, and parameter uncertainties, thereby simulating realistic operating conditions.

3.4. Fuzzy Logic Controller Schematic

The Fuzzy Logic Controller (FLC) schematic for the inverted pendulum and cart system is composed of several interconnected subsystems that collectively maintain the pendulum in its upright position while stabilizing the cart's motion. The overall model integrates the fuzzy inference process, gain normalization, and the nonlinear pendulum–cart dynamics into a closed-loop control framework (Figure 4).

The input signals, including the pendulum's angular displacement (θ), angular velocity ($\dot{\theta}$), cart position (x), and cart velocity (\dot{x}), are first normalized through gain scaling blocks. Specifically, the normalization factors are set as $K_1 = 1/(\pi/6)$, $K_2 = 1/2$, $K_3 = 1/3$, and $K_4 = 1/4$, ensuring that each state variable is mapped into a standard range $[-1,1]$ suitable for fuzzy computation. These scaled values are then fed into the fuzzy logic controller block, which represents the rule-based decision-making core of the system.

Within the FLC, the input variables are processed using predefined fuzzy membership functions and rule sets that determine the control signal based on the system's state deviations. The controller evaluates the nonlinear interaction between the pendulum's angular behavior and the cart's translational motion, producing a normalized control output \bar{u} . This output is subsequently amplified by a gain $K_5 = 120$ to generate the actual control force u , which acts on the cart to influence its acceleration and restore the pendulum to equilibrium.

The “Pole and Cart” dynamic block models the nonlinear behavior of the coupled system, where the control input u affects both the rotational dynamics of the pendulum and the translational motion of the cart. The outputs, $\theta, \dot{\theta}, x, \dot{x}$, are continuously fed back into the fuzzy controller, forming a real-time feedback loop. This feedback mechanism enables adaptive correction of the control force, ensuring that the pendulum remains balanced and the cart's position is stabilized under dynamic and disturbance conditions.

The simulation of the inverted pendulum and cart system is carried out in MATLAB/Simulink to evaluate the performance and robustness of the proposed fuzzy logic controller. The objective of the simulation is to maintain the pendulum in an upright position while simultaneously stabilizing the cart within a predefined range, despite the presence of nonlinearities and external disturbances.

The Simulink model incorporates the mathematical dynamics of the inverted pendulum derived from the nonlinear coupled equations of motion. The fuzzy logic controller is embedded within the control loop, providing the adaptive force command based on real-time state feedback. The simulation operates under continuous-time integration, ensuring accurate representation of the pendulum's oscillatory and transient responses.

To assess controller effectiveness, the simulation parameters are initialized to represent a typical inverted pendulum setup with realistic physical constants such as mass, length, and friction coefficients. The input force u is constrained within the range $[-40,40]$ N to reflect actuator limitations, while state variables - pendulum angle (θ), angular velocity ($\dot{\theta}$), cart position (x), and velocity (\dot{x}), are measured and fed back into the fuzzy controller.

The simulation environment allows for monitoring of both the pendulum's stabilization response and the cart's displacement over time. Performance indicators such as settling time, overshoot, and steady-state error are analyzed to quantify the controller's precision and adaptability. Furthermore, external disturbances and parameter variations are introduced to evaluate the controller's robustness under uncertain conditions.

Through this simulation setup, the FLC's ability to effectively manage the system's nonlinear dynamics and maintain balance stability is demonstrated, confirming its potential as a reliable control solution for underactuated systems like the inverted pendulum.

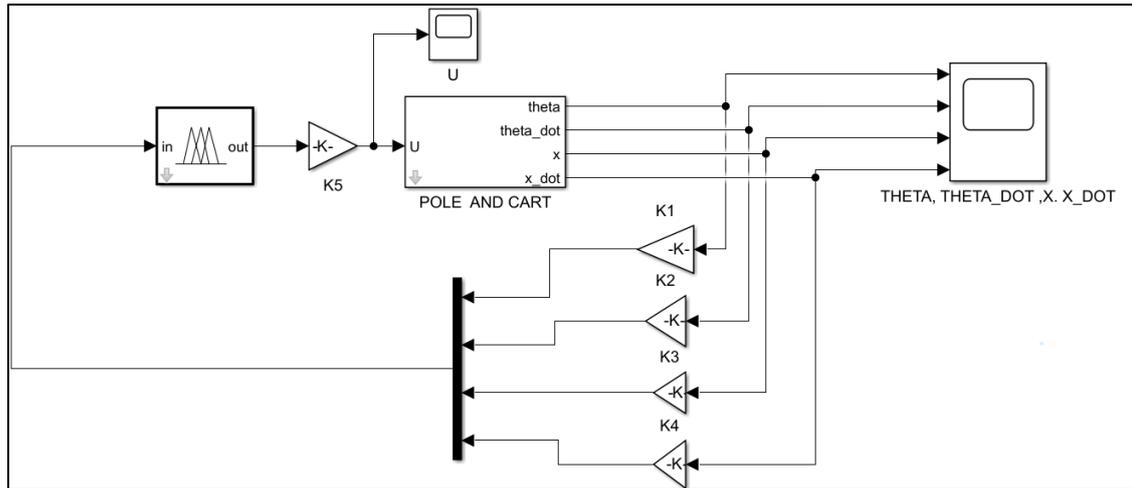


Figure 5 Fuzzy logic controller for Inverted Pendulum and Cart system

4. Experience result

The simulation of the Ball and Beam system was performed using a fuzzy logic controller to regulate the ball's position along the beam. The system was evaluated under different initial conditions to assess its stability and response characteristics.

Simulation Results for Initial Condition $\theta_0 = 0.4, \dot{\theta}_0 = 0.1, x_0 = -0.5; \dot{x}_0 = 0$

The simulation results clearly demonstrate the effectiveness of the proposed Fuzzy Logic Controller (FLC) in stabilizing the inverted pendulum–cart system (Figure 5). The pendulum angle (θ) initially deviates significantly from the vertical position but is rapidly corrected, reaching a stable upright equilibrium within less than 5 seconds. The quick settling response highlights the controller's ability to handle the system's inherent nonlinearity and strong coupling efficiently.

The angular velocity ($\dot{\theta}$) and cart position (x) exhibit smooth transient responses with minimal overshoot, while the velocity of the cart (\dot{x}) stabilizes promptly, indicating that the FLC effectively suppresses oscillations and prevents instability. Moreover, the control signal (u) shows an initial large corrective effort followed by a gradual decay to zero, confirming that the system achieves balance rapidly and with minimal control energy.

Overall, the results confirm that the proposed FLC achieves fast stabilization (settling time < 5 s), robust damping, and steady-state accuracy for the nonlinear and underactuated inverted pendulum–cart system. This demonstrates a significant improvement in transient performance and control smoothness compared with traditional PID-based approaches.

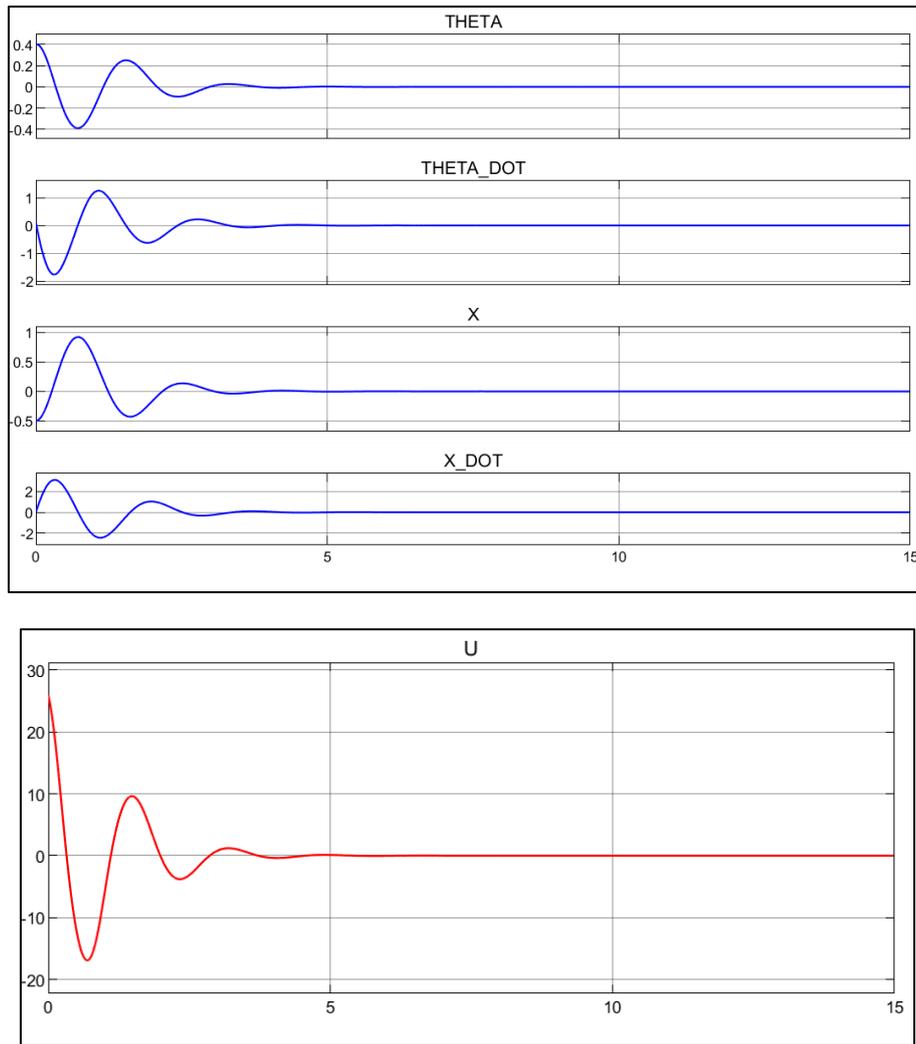


Figure 6 Control Signal and Respond of system in Simulation 1

Simulation Results for Initial Condition $\theta_0 = -0.3, \dot{\theta}_0 = -0.2, x_0 = 0; \dot{x}_0 = 0.1$

The simulation results demonstrate that the fuzzy logic controller (FLC) effectively stabilizes the inverted pendulum–cart system from a negatively inclined initial condition (Figure 6). The pendulum angle (θ) initially exhibits a noticeable overshoot due to the system’s nonlinear coupling but rapidly converges toward the upright equilibrium. The stabilization occurs within approximately 5 seconds, indicating the controller’s capability for fast settling and efficient compensation of nonlinear dynamics.

The angular velocity ($\dot{\theta}$) and cart motion (x, \dot{x}) responses confirm the smooth convergence process, with oscillations decaying rapidly and no sustained instability observed. The cart’s velocity (\dot{x}) also settles quickly, reflecting effective damping behavior and robust control action. Meanwhile, the control signal (u) shows an initial large corrective effort followed by a rapid decline to zero, suggesting that the controller achieves equilibrium efficiently without excessive control energy.

Overall, these results verify that the proposed FLC ensures stable and fast recovery (settling time < 5 s) even under adverse initial conditions. The system remains well-damped and free from residual oscillations, highlighting the robustness and adaptability of the fuzzy controller in handling the nonlinear dynamics of the inverted pendulum–cart system.

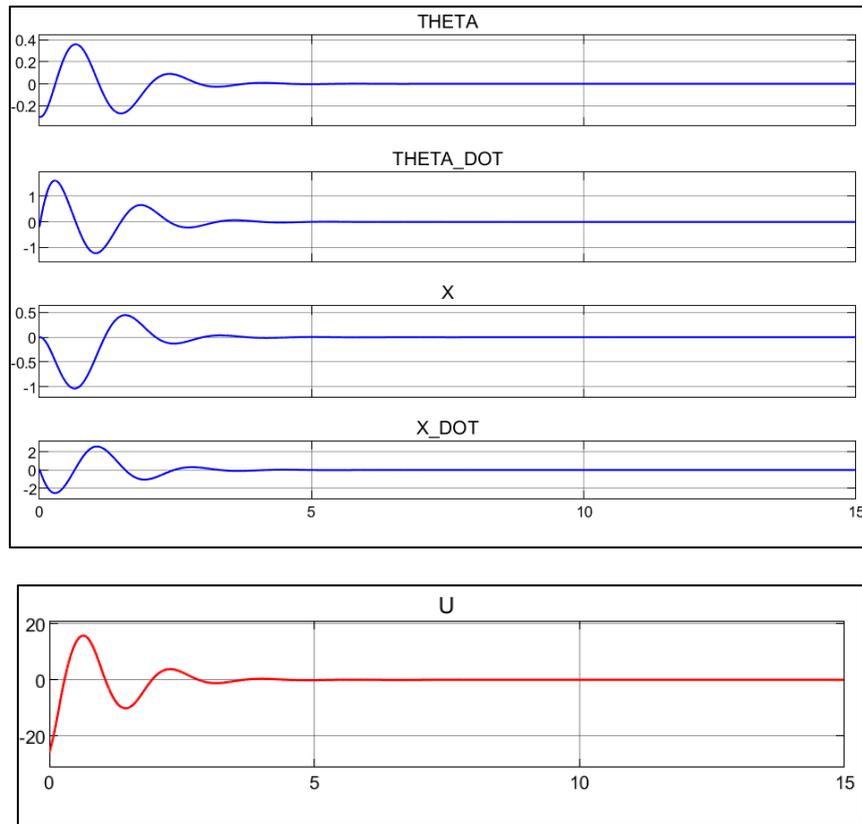


Figure 7 Control Signal and Respond of system in Simulation 2

5. Conclusion

This study presented the design and implementation of a Fuzzy Logic Controller (FLC) for stabilizing the inverted pendulum and cart system, a classical benchmark of nonlinear and underactuated dynamics. The proposed control strategy successfully handled the system's nonlinearities, strong coupling, and inherent instability without requiring linearization or precise modeling. Through the development of a mathematical model, normalization of input and output variables, and formulation of rule-based fuzzy inference, the controller demonstrated excellent capability in maintaining upright balance and minimizing oscillations.

Simulation results verified that the fuzzy controller achieved fast stabilization with a settling time of less than 5 seconds, low overshoot, and smooth convergence for various initial conditions. The FLC effectively regulated both the pendulum's angular displacement and the cart's translational motion while ensuring efficient energy utilization. Moreover, the controller exhibited strong robustness and adaptability, maintaining stable operation even under nonlinear effects, parameter variations, and external disturbances.

Overall, the results confirm that fuzzy logic control provides a robust, adaptive, and interpretable solution for balancing nonlinear systems such as the inverted pendulum on a cart. Unlike conventional PID or linear control methods, the proposed FLC offers greater flexibility and resilience to uncertainties. Future work will focus on extending this framework to real-time hardware implementation, integrating optimization-based tuning of membership functions, and exploring hybrid fuzzy–reinforcement learning schemes to further enhance performance in real-world environments.

Compliance with ethical standards

Disclosure of conflict of interest

No conflict of interest to be disclosed.

Statement of informed consent

Informed consent was obtained from all individual participants included in the study.

References

- [1] Tang, W., & Ahmad, I. (2024). Fuzzy logic approach for controlling uncertain and nonlinear systems: A comprehensive review of applications and advances. *International Journal of Modelling and Simulation*, 44(3), 215–232.
- [2] Melese, G., Lee, S., & Yoon, H. (2025). Optimal fuzzy–PID design for nonlinear object tracking systems using evolutionary algorithms. *Scientific Reports*, 15, 92309.
- [3] Su, Xiaojie, et al. "Output feedback fuzzy control of nonlinear dynamic systems: Event-triggered case." *International Journal of Robust and Nonlinear Control* 31.14 (2021): 6527-6548.
- [4] Liu, Ruixia, et al. "Adaptive fuzzy command filtered control for uncertain fractional order nonlinear systems with full state constraints." *Scientific Reports* 15.1 (2025): 4615.
- [5] Al-Hadithi, Basil Mohammed, and Gilberth André Loja Acuña. "Control of Nonlinear Systems Using Fuzzy Techniques Based on Incremental State Models of the Variable Type Employing the "Extremum Seeking" Optimizer." *Applied Sciences* 15.14 (2025): 7791.
- [6] Castillo, Oscar, and Patricia Melin. "A new fuzzy fractal control approach of non-linear dynamic systems: The case of controlling the COVID-19 pandemics." *Chaos, Solitons & Fractals* 151 (2021): 111250.
- [7] Bao, Dan, et al. "A framework of adaptive fuzzy control and optimization for nonlinear systems with output constraints." *Information Sciences* 616 (2022): 411-426.
- [8] Lamamra, Kheireddine, Farida Batat, and Fouad Mokhtari. "A new technique with improved control quality of nonlinear systems using an optimized fuzzy logic controller." *Expert Systems with Applications* 145 (2020): 113148.
- [9] Roose, A., & Yahya, M. (2017). Fuzzy-logic control of an inverted pendulum on a cart. *Computers & Electrical Engineering*, 64, 233–248.
- [10] Mahmoodabadi, M. J., Rahmani, H., & Hooshmand, R. (2020). A new optimal robust adaptive fuzzy controller for a class of systems including the inverted pendulum system. *International Journal of Modelling and Simulation*, 40(5), 331–341.
- [11] Nguyen, T. V., Pham, H. T., & Lee, S. H. (2023). Enhancing stability control of inverted pendulum using Takagi–Sugeno fuzzy model with disturbance rejection and input–output constraints. *Scientific Reports*, 13, 14129.