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(Research Article)

On Generalized Jordan (σ,τ)-k-Homomorphism of Γ-rings

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Abstract

Assume N be a Γ-ring with N be a Γ́-ring. In the presented paper introduced the concept of generalized (σ,τ)-k-homomorphism , generalized Jordan (σ,τ)-k-homomorphism, generalized Jordan triple (σ,τ)-k-homomorphism generalized(σ,τ)-k–anti homomorphism with show that under certain conditions of N, any generalized Jordan (σ,τ)-k-homomorphism F from a Γ-ring N into a two-torsion free Γ́ -ring N, such that $w\alpha r\beta w = w\beta r\alpha w$, for any $w, r \in$ N with $\alpha, \beta \in \Gamma$, $w'\alpha' r'\beta' w' = w'\beta' r'\alpha' w'$, for any $w', r' \in$ Ń with α', β'∈ Γ́ , $\sigma^2 = \sigma$, $\tau^2 = \tau$ with $\sigma\tau = \tau\sigma$, then D is a generalized Jordan triple (σ,τ)-k-homomorphism.

Keywords: Γ-Ring; Generalized Homomorphism; Generalized Jordan Homomorphism

1. Introduction

Assume N with Γ be 2- additive abelian groups, assume that there is a mapping from

$N\times\Gamma\times N \longrightarrow N$ (the image of (w,α,k) being symbolized by $w\alpha k$, $w, k \in N$ with $\alpha\in\Gamma$) satisfying for any $w, k, h \in N$ with α, $\beta \in \Gamma$

(i) $(w + k)\alpha h = w\alpha h + k\alpha h$

$w(\alpha + \beta)h = w\alpha h + w\beta h$

$w\alpha(k + h) = w\alpha k + w\alpha h$

(ii) $(w\alpha k)\beta h = w\alpha(k\beta h)$, [1-3] .

A Γ-ring N is named a prime if $w\Gamma N\Gamma k = (0)$ leads to $w=0$ or $k=0$, where $w, k \in N$, [2-4].

A Γ-ring N is named semi-prime if $w\Gamma N\Gamma w = (0)$ leads to $w=0$, suchh that $w \in N$, ... [2-4].

Assume N be a two-torsion free semi-prime Γ-ring with assume that $w, k\in N$ if $w\Gamma n\Gamma k + k\Gamma n\Gamma w = 0$ for any $n \in N$, then $w\Gamma n\Gamma k = k\Gamma n\Gamma w = 0$ [5-6] .

Assume N be Γ-ring then N is named two-torsion free if $2w = 0$ leads to $w = 0$, for any $w \in N$, [5]

Assume θ be an additive mapping of a ring R into a ring Ŕ, θ is named a homomorphism if

$\theta(w k) = \theta(w)\,\theta(k)$.

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with θ is named a Jordan homomorphism if for any w, ķ ∈ Ɍ

$\theta(w ķ + ķ w) = \theta(w)\,\theta(ķ) + \theta(ķ)\,\theta(w)$ for any w, ķ ∈ Ɍ , [2] .

An additive mapping D of a ring R into a ring Ŕ is said to be a generalized homomorphism if there exist a homomorphism θ from a **r**ing R into a **r**ing Ŕ , such that for any $w, k \in$ Ɍ

D $(wbk) = D(w)\theta(k)$ for any w , $k \in$ Ɍ, where θ is named the relating homomorphism with D is named a generalized Jordan homomorphism if there exist a Jordan homomorphism θ from a **r**ing R into a **r**ing Ŕ suchḥ that for any $w, k \in$ Ɍ

D $(wk + kw) = D(w)\theta(k) + D(k)\,\theta(w)$,where θ is named the relating Jordan homomorphism [7].

Assume θ be an additive mapping of a Γ-**r**ing N into a Γ́-**r**ing Ń, θ is named homomorphism

if $\theta(w\alpha k) = \theta(w)\alpha\theta(k)$, for any $w, k \in$ N with $\alpha \in \Gamma$. [1].

Assume θ be an additive mapping of Γ-ring N into a Γ́-ring Ń, θ is named Jordan homomorphism if $\theta(w\alpha k + k\alpha w) = \theta(w)\alpha\theta(k) + \theta(k)\alpha\theta(w)$, for any $w, k \in$ N with $\alpha \in \Gamma$, [6].

An additive mapping D of a Γ-ring N into a Γ́-ring Ń is named generalized homomorphism if there exists a homomorphism θ from a Γ-ring N into a Γ́ -ring΄N, suchḥ that for any $w, k \in$ N with $\alpha \in \Gamma$, D $(w\alpha k) = D(w)\alpha\theta(k)$, where θ is named the Ɍelating homomorphism with D is named a generalized Jordan homomorphism if there exists a Jordan homomorphism θ from a Γ-ring N into a Γ́-ring N', suchḥ that for any $w, k \in$ N with $\alpha \in \Gamma$

D $(w\alpha k + k\alpha w) = D(w)\alpha\theta(k) + D(k)\alpha\theta(w)$ [6].

Assume θ be an additive mapping of a ring R into a ring R' withσ, τ be 2- endomorphism of R. θ is named (σ,τ)-homomorphism if $\theta(wk) = \theta(\sigma(w))\,\theta(\tau(k))$, for any $w, k \in$ R with θ is named Jordan (σ,τ)-homomorphism if $\theta(wk + kw) = \theta(\sigma(w))\,\theta(\tau(k)) + \theta(\sigma(k))\,\theta(\tau(w))$, for any $w, k \in$ Ɍ [3].

Assume D be an additive mapping of a ring R into a ring R' with σ, τ be 2- endomorphism of R .

D is named a generalized (σ,τ)-homomorphism if there exists a (σ,τ)-homomorphism θ from a ring R into a ring R ' suchḥ that, D $(wk) = D(\sigma(w))\,\theta(\tau(k))$, for any $w, k \in$ R .

Where θ is named the Relating (σ,τ)-homomorphism with D is named a generalized Jordan (σ,τ)-homomorphism if there exists a Jordan (σ,τ)-homomorphism θ from a ring R into a ring Ɍ' such that, $D(wk + kw) = D(\sigma(w))\,\theta(\tau(k)) + D(\sigma(k))\,\theta(\tau(w))$, for any $w, k \in$ R .

Where θ is named tḥe Ɍelating (σ,τ)-homomorphism [3] .

Assume θ be an additive mapping of a Γ-ring N into a Γ́ -ring N' withσ, τ be 2-endomorpḥisms of N. θ Is named (σ,τ)-homomorphism if $\theta(w\alpha r) = \theta(\sigma(w))\alpha\theta(\tau(r))$, for any $w, u \in$ N with $\alpha \in \Gamma$ with θ is named Jordan (σ,τ)-homomorphism if $\theta(w\alpha k + k\alpha w) = \theta(\sigma(w))\alpha\theta(\tau(k)) + \theta(\sigma(k))\alpha\theta(\tau(w))$, for any $w, k \in$ N with $\alpha \in \Gamma$ [3] .

Assume F be an additive mapping of a Γ-ring N into a Γ́-ring Ń with σ, τ be 2 - endomorphism of N . D is named a generalized (σ,τ)-homomorphism if there exists a (σ,τ)-homomorphism θ from a Γ-ring N into a Γ́ -ring Ń suchḥ that

D $(w\alpha k) = D(\sigma(w))\alpha\theta(\tau(k))$, for any $w, k \in$ N with $\alpha \in \Gamma$.

Where θ is named the relating (σ,τ)-homomorphism with D is named a generalized Jordan (σ,τ)-homomorphism if there exist a Jordan (σ,τ)-homomorphism θ from a Γ- ring N into a Γ́ - ring Ń such that

D $(w\alpha\, k + k\,\alpha w) = D(\sigma(w))\alpha\theta(\tau(u)) + D(\sigma(k))\alpha\theta(\tau(w))$, for any $w, k \in$ N with $\alpha \in \Gamma$.

Where θ is named the relating Jordan (σ,τ)-homomorphism [3] .

Now, the main purpose of this paper is that any generalized Jordan (σ,τ)-k-homomorphism of a Γ-ring N into a prime Γ́ -ring Ń is either generalized (σ,τ)-k-homomorphism or (σ,τ)-k -anti homomorphism.

2. Methodology, Generalized (σ,τ)-k-Homomorphism of Γ-rings

We use an algebraic and structural method to study the generalized Jordan (σ,τ)-k-homomorphism of a Γ-ring in this study. The basic steps used in the methodology are:

2.1. Definitional Framework

We start by defining and optimizing some important algebraic structures, such as Γ-rings, generalized (σ,τ)-k-homomorphism, generalized (σ,τ)-k-homomorphism, and generalized (σ,τ)-k-homomorphism. Those generalizations are due to these definitions that generalize classical concepts of rings to that of more general structure of Γ-rings where endomorphism σ and τ are commuting [2-5].

2.2. Act utilization of Functional Identities

The use of mapping properties and functional identities are central towards our approach. We set δ(w, r)_α = D(wαr) − D(σ(w))k(α)θ(τ(r)) to measure the failure of D to be a generalized (σ, τ)-k-homomorphism. This device enables us to re-examine methodically in different cases of rings the habits of D [4-7].

2.2.1. Assumptions on the Structure of the Algebra

We are operating under the following structural assumptions: N and Ń being two-torsion free, Ń being prime or semi-prime,

$\sigma^2 = \sigma$, $\tau^2 = \tau$, and $\sigma\tau = \tau\sigma$,

Commutativity-type properties: $w\alpha r\beta w = w\beta r\alpha w$

2.3. Proof Techniques

The proofs rely on:

Algebraic manipulation of generalized Jordan conditions, Substitution and expansion in Γ–rings. Annihilation properties deduced by use of primness and two-torsion freeness, Inductive and combinatorics evidence to generalize results of Jordan to Jordan triple homomorphism.

Comparative Reasoning: This involves using reason to make comparisons of things. We often make comparisons between the terms formed with the help of various kinds of homomorphism (Jordan vs. triple) and apply the inductive substitution of the variables to extend them. The conclusion that some products are forced to be zero is made with the help of the prime condition which results in the dichotomy of the main theorem [5-8].

2.4. Definition

Assume D be an additive mapping of a Γ-ring N into a Γ́ -ring Ń with K be an additive mapping of Γ into Γ́ , σ, τ be 2 endomorphism of N . D is named a generalized (σ,τ)-k-homomorphism if there exist a (σ,τ)-k-homomorphism θ from a Γ-ring N into a Γ́ -ring Ń such that

$D(w\alpha r) = D(\sigma(w))\, k(\alpha)\theta(\tau(r))$, for any $w, r \in N$ with $\alpha \in \Gamma$.

Where θ is named the Relating (σ,τ)-k-homomorphism.

2.5. Example

Assume R be a ring .Assume $N = N_{1\times 2}(R)$, $Ń = N_{1\times 2}(R)$ with

$$\Gamma = \left\{ \begin{pmatrix} n \\ 0 \end{pmatrix}, n \in Z \right\}$$

Then N with Ń are 2 -Γ-ring.

We use the usual addition with multiplication on matrices of N×Γ×N. Assume $k = (k_i)_{i\in N}$ be a family of additive mapping from Γ into itself. Assume D be an additive mapping of a Γ-ring N into a Γ́ -ring Ń , such that

$D((w \quad k)) = (w \quad o)$, for any $(w \quad k) \in N$.

Then there exists a (σ,τ)-homomorphism θ from a Γ-ring N into a Γ́ -ring Ń, suchh that,

$\theta((w \quad k)) = (-w \quad o)$, for any $(w \quad k) \in N$.

Assume σ, τ be 2- endomorphism of M, such that ,

$\sigma((w \quad k)) = (w \quad k)$, $\tau((w \quad k)) = (-w \quad -k)$.

Then D is generalized (σ,τ)-homomorphism.

2.6. Definition

Assume D be an additive mapping of a Γ-ring N into a Γ́-ring Ń with Ķ be an additive mapping of Γ into Γ',σ, τ be 2 - endomorphism of N. D is named generalized Jordan (σ,τ)-k-homomorphism if there exist a Jordan (σ,τ)-k-homomorphism θ from a Γ- ring N into a Γ'- ring Ń such that,

$D(w\alpha k + k\alpha w) = D(\sigma(w))\alpha\theta(\tau(k)) + D(\sigma(k))\alpha\theta(\tau(w))$, for any $w, k \in N$ with $\alpha \in \Gamma$.

Where θ is named the Relating Jordan (σ,τ)-k-homomorphism.

2.7. Definition

Assume D be an additive mapping of a Γ-ring N into a Γ́-ring Ń with K be an additive mapping of Γ into Γ́, σ, τ be 2-endomorphism of N . D is named generalized Jordan triple (σ,τ)-k -homomorphism if there exists a Jordan triple (σ,τ)-k-homomorphism θ from a Γ-ring N into a Γ́ -ring Ń , such that,

$D(w\alpha r\beta w) = D(\sigma(w))k(\alpha)\theta(\sigma\tau(r))\, k(\beta)\theta(\tau(w))$, for any $w, r \in N$ with $\alpha, \beta \in \Gamma$.

Where θ is named the Relating Jordan triple (σ,τ)-k -homomorphism.

2.8. Definition

Assume D be an additive mapping of a Γ-ring N into a Γ́ -ring Ń with K be an additive mapping of Γ into Γ́, σ, τ be 2-endomorphism of N . D is named generalized (σ,τ)-k-anti homomorphism if there exists w (σ,τ)-k-anti homomorphism from a Γ-ring N into a Γ́ -ring N' such that

$D(w\alpha r) = D(\sigma(r))\, k(\alpha)\theta(\tau(w))$, for any $w, r \in N$ with $\alpha \in \Gamma$.

Where θ is named the Relating (σ,τ)-k-anti homomorphism.

2.9. Lemma

Assume D be a generalized Jordan triple (σ,τ)-k-homomorphism of a Γ-ring N into a Γ́ -ring Ń, then for any $w, r, v \in N$ with $\alpha, \beta \in \Gamma$

If $\sigma^2 = \sigma$, $\tau^2 = \tau$ with $\sigma\tau = \tau\sigma$, then

$D(w\alpha r\beta w + w\beta r\alpha w) = D(\sigma(w))\, k(\alpha)\theta(\sigma\tau(r))\, k(\beta)\theta(\tau(w)) + D(\sigma(w))\, k(\beta)\theta(\sigma\tau(r))\, k(\alpha)\theta(\tau(w))$

(ii) $D(w\alpha r\beta v + v\alpha r\beta w) = D(\sigma(w))\, k(\alpha)\theta(\sigma\tau(r))\, k(\beta)\theta(\tau(v)) + D(\sigma(v))\, k(\alpha)\theta(\sigma\tau(r))\, k(\beta)\theta(\tau(w))$

(iii) In particular, if N, Ń be two commutative with Ń is a two-torsion free, then

$D(w\alpha r\beta v) = D(\sigma(w))\, k(\alpha)\theta(\sigma\tau(r))\, k(\beta)\theta(\tau(v))$

(**iv**) $D(w\alpha r\alpha v + v\alpha r\alpha w) = D(\sigma(w))\, k(\alpha)\theta(\sigma\tau(r))\, k(\alpha)\theta(\tau(v)) + D(\sigma(v))\, k(\alpha)\theta(\sigma\tau(r))\, k(\alpha)\theta(\tau(w))$

Proof:

(**i**) Replace $w\beta r + r\beta w$ for r in Definition (2.3), we obtain:

$D(w\alpha(w\beta r + r\beta w) + (w\beta r + r\beta w)\alpha w) = D(\sigma(w))\ k\ (\alpha)\theta(\tau(w\beta r + r\beta w)) + D(\sigma(w\beta r + r\beta w))\ k\ (\alpha)\theta(\tau(w))$

$= D(\sigma(w))\ k\ (\alpha)\theta(\tau(w)\beta\tau(r)+\tau(r)\beta\tau(w))+D(\sigma(w)\beta\sigma(r)+\sigma(r)\beta\sigma(w))\ k\ (\alpha)\theta(\tau(w))$

$= D\ (\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(w))\ k\ (\beta)\theta(\tau^2(r))+D(\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau^2(w)) +$

$D(\sigma^2(w))\ k\ (\beta)\theta(\tau\sigma(r))\ k\ (\alpha)\theta(\tau(w))+D(\sigma^2(r))\ k\ (\beta)\theta(\tau\sigma(w))\ k\ (\alpha)\theta(\tau(w))$

Since $\sigma^2 = \sigma$, $\tau^2 = \tau$ with $\sigma\tau = \tau\sigma$

$= D\ (\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(w))\ k\ (\beta)\theta(\tau(r))+D(\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(w)) +$

$D\ (\sigma(w))\ k\ (\beta)\theta(\sigma\tau(r))\ k\ (\alpha)\theta(\tau(w))+D(\sigma(r))\ k\ (\beta)\theta(\sigma\tau(w))\ k\ (\alpha)\theta(\tau(w))$ …… (1)

In contrast

$D(w\alpha(w\beta r + u\beta w)+(w\beta r + r\beta w)\alpha w)=D(w\alpha w\beta r + w\alpha r\beta w+ w\beta r\alpha w+ r\beta w\alpha w)= D(\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(\mathrm{w}))\ k\ (\beta)\theta(\tau(r))$

$+D\ (\sigma(r))k(\beta)\theta(\sigma\tau(w))\ k\ (\alpha)\theta(\tau(r))+D(w\alpha r\beta w+ w\beta r\alpha w)$ …….(2)

Evaluate (1) with (2), we obtain:

$D\ (w\alpha r\beta w+ w\beta r\alpha w)=D(\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(w))+D(\sigma(w))\ k\ (\beta)\theta(\sigma\tau(r))\ k\ (\alpha)\theta(\tau(w))$

Replace $w + v$ for w in Definition (2.4), we obtain:

$D\ ((w + v)\alpha r\beta(w + v)) = D(\sigma(w + v))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(w + v))$

$= D\ (\sigma(w) + \sigma(v))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(w) + \tau(v))$

$= D\ (\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(w))+D(\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(v)) +$

$D\ (\sigma(v))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(w))+D(\sigma(v))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(v))$ ……(1)

In contrast

$D\ ((w + v)\alpha u\beta(w + v)) = D(w\alpha r\beta w + w\alpha r\beta v + v\alpha r\beta w + v\alpha r\beta v)$

$=D(\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(w))+D(\sigma(v))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(v))+D(w\alpha r\beta v+ v\alpha r\beta w)$ ……(2)

Evaluate (1) with (2), we obtain:

$D(w\alpha r\beta v+ v\alpha r\beta w) = D(\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(v))+D(\sigma(v))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(w))$

(**iii**)By (ii) with since N, Ń be 2- commutative with Ń is a two-torsion free

$D\ (w\alpha r\beta \mathrm{v} + w\alpha r\beta \mathrm{v}) = 2D(w\alpha r\beta \mathrm{v}) = D(\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\beta)\theta(\tau(v))$

(**iv**)Replace α for β in (ii), we obtain:

$D(w\alpha r\alpha v + v\alpha r\alpha w)=D(\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\alpha)\theta(\tau(v))+D(\sigma(v))\ k\ (\alpha)\theta(\sigma\tau(r))\ k\ (\alpha)\theta(\tau(w))$

2.10. Definition

Assume D be a generalized Jordan (σ,τ)-k-homomorphism of a Γ-ring N into a Γ́ -ring Ń, then for any w, $u \in$ N with $\alpha \in \Gamma$, we define δ :N×Γ×N⟶ Ń by $\delta(w,r)_\alpha = D(w\alpha r) - D(\sigma(w))\ k\ (\alpha)\theta(\tau(r))$.

2.11. Lemma

If D is a generalized Jordan (σ,τ)-k-homomorphism of a Γ-ring N into a Γ́ -ring Ń, then for any

$w, r, v \in$ N with $\alpha, \beta \in \Gamma$

(i) $\delta(w, r)_\alpha = -\delta(r, w)_\alpha$

(ii) $\delta(w + r, v)_\alpha = \delta(w, v)_\alpha + \delta(r, v)_\alpha$

(iii) $\delta(w, r + v)_\alpha = \delta(w, r)_\alpha + \delta(w, v)_\alpha$

(iv) $\delta(w, r)_{\alpha+\beta} = \delta(w, r)_\alpha + \delta(w, r)_\beta$

Proof:

i) By 2.3: Definition

$D(w\alpha r + r\alpha w) = D(\sigma(w))\ k\ (\alpha)\theta(\tau(r)) + D(\sigma(r))\ k\ (\alpha)\theta(\tau(w))$

$D(w\alpha r) - D(\sigma(w))\ k\ (\alpha)\theta(\tau(r)) = -(D(r\alpha w) - D(\sigma(r))\ k\ (\alpha)\theta(\tau(w)))$

$\delta(w,r)_\alpha = -\delta(r, w)_\alpha$

ii) $\delta(w + r, v)_\alpha = D((w + r)\alpha v) - D(\sigma(w + r))\ k\ (\alpha)\theta(\tau(v))$

$= D(w\alpha v + r\alpha v) - D(\sigma(w))\ k\ (\alpha)\theta(\tau(v)) - D(\sigma(r))\ k\ (\alpha)\theta(\tau(v))$

$= D(w\alpha v) - D(\sigma(w))\ k\ (\alpha)\theta(\tau(v)) + D(u\alpha v) - D(\sigma(r))\ k\ (\alpha)\theta(\tau(v))$

$= \delta(w, v)_\alpha + \delta(u, v)_\alpha$

(iii) $\delta(w, r + v)_\alpha = D(w\alpha(r+ v)) - D(\sigma(w))k(\alpha)\theta(\tau(r + v))$

$= D(w\alpha r + w\alpha v) - D(\sigma(w))\ k\ (\alpha)\theta(\tau(r)) - D(\sigma(w))\ k\ (\alpha)\theta(\tau(v))$

$= D(w\alpha r) - D(\sigma(w))\ k\ (\alpha)\theta(\tau(r)) + D(w\alpha v) - D(\sigma(w))\ k\ (\alpha)\theta(\tau(v))$

$= \delta(w,r)_\alpha + \delta(w,v)_\alpha$

iv) $\delta(w, r)_{\alpha+\beta} = D(w(\alpha + \beta)r) - D(\sigma(w))\ k\ (\alpha + \beta)\theta(\tau(r))$

$= D(w\alpha r) - D(\sigma(w))\ k\ (\alpha)\theta(\tau(r)) + D(w\beta r) - D(\sigma(w))\ k\ (\beta)\theta(\tau(r)) = \delta(w, br)_\alpha + \delta(w,r)_\beta$

2.12. Remark

Note that D is a generalized (σ,τ)-k-homomorphism of a Γ-ring N into a Γ́ -ring Ń if with only if $\delta(w,r)_\alpha = 0$ for any w, $r \in$ N with $\alpha \in \Gamma$.

2.13. Lemma

Assume D be a generalized Jordan (σ,τ)-k-homomorphism of a Γ-ring N into a Γ́ -ring Ń, such that $\sigma^2 = \sigma$, $\tau^2 = \tau$ with $\sigma\tau = \tau\sigma$, then for any w, r, v ∈ N with $\alpha, \beta \in \Gamma$

(i) $\delta(\sigma(w),\sigma(r))_\alpha\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)G(\tau(r),\tau(w))_\alpha + \delta(\sigma(r),\sigma(w))_\alpha\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)G(\tau(w),\tau(r))_\alpha = 0$

(ii) $\delta(\sigma(w),\sigma(r))_\alpha\ k\ (\alpha)\theta(\sigma\tau(v))\ k\ (\alpha)G(\tau(r),\tau(w))_\alpha + \delta(\sigma(r),\sigma(w))_\alpha\ k\ (\alpha)\theta(\sigma\tau(v))\ k\ (\alpha)G(\tau(w),\tau(r))_\alpha = 0$

(iii) $\delta(\sigma(w),\sigma(r))_\beta\ k\ (\alpha)\theta(\sigma\tau(v))\ k\ (\alpha)G(\tau(r),\tau(w))_\beta + \delta(\sigma(r),\sigma(w))_\beta\ k\ (\alpha)\theta(\sigma\tau(v))\ k\ (\alpha)G(\tau(w),\tau(r))_\beta = 0$

Proof:

(i)Assume w = $w\alpha r\beta v\beta r\alpha w + r\alpha w\beta v\beta w\alpha r$, since D is a generalized Jordan (σ,τ)-k-homomorphism

$$D(t) = D(w\alpha(r\beta v\beta r)\alpha w + r\alpha(w\beta v\beta w)\alpha r)$$

$$= D(\sigma(w))\ k\ (\alpha)\theta(\sigma\tau(r\beta v\beta r))\ k\ (\alpha)\theta(\tau(w)) + D(\sigma(r))\ k\ (\alpha)\theta(\sigma\tau(w\beta v\beta w))\ k\ (\alpha)\theta(\tau(r))$$

$$= D(\sigma(w))\ k\ (\alpha)\theta(\sigma(\sigma\tau(r)))\ k\ (\beta)\theta(\sigma\tau(\sigma\tau(v)))\ k\ (\beta)\theta(\tau(\sigma\tau(r)))\ k\ (\alpha)\theta(\tau(w)) +$$

$$D(\sigma(r))\ k\ (\alpha)\theta(\sigma(\sigma\tau(w)))\ k\ (\beta)\theta(\sigma\tau(\sigma\tau(v)))\ k\ (\beta)\theta(\tau(\sigma\tau(w)))\ k\ (\alpha)\theta(\tau(r)) \quad \ldots\ldots(1)$$

In contrast

$$D(t) = D((w\alpha r)\beta n\beta(r\alpha w) + (r\alpha w)\beta n\beta(w\alpha r))$$

$$= D(\sigma(w\alpha r))\ k\ (\beta)\theta(\sigma\tau(n))\ k\ (\beta)\theta(\tau(r\alpha w)) + D(\sigma(r\alpha w))\ k\ (\beta)\theta(\sigma\tau(n))\ k\ (\beta)\theta(\tau(w\alpha r))$$

$$D(t) = D(\sigma(w\alpha r))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)(\theta(\sigma\tau(w))\ k\ (\alpha)\theta(\tau^2(r)) + \theta(\sigma\tau(r))\ k\ (\alpha)\theta(\tau^2(w)) -$$

$$\theta(\tau(w\alpha r))) + (-D(\sigma(w\alpha r)) + D(\sigma^2(w))\ k\ (\alpha)\theta(\tau\sigma(r)) +$$

$$D(\sigma^2(r))\ k\ (\alpha)\theta(\tau\sigma(w)))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)\theta(\tau(w\alpha r))$$

$$= -D(\sigma(w\alpha r))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)(\theta(\tau(w\alpha r)) - \theta(\sigma\tau(w))\ k\ (\alpha)\theta(\tau^2(r)) -$$

$$D(\sigma(w\alpha r))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)(\theta(\tau(w\alpha r)) - \theta(\sigma\tau(r))\ k\ (\alpha)\theta(\tau^2(w))) +$$

$$D(\sigma^2(w))\ k\ (\alpha)\theta(\tau\sigma(r))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)\theta(\tau(w\alpha r)) + D(\sigma^2(r))\ k\ (\alpha)\theta(\tau\sigma(w))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)\theta(\tau(w\alpha r)) \quad \ldots\ldots(2)$$

Evaluate (1), (2) with since $\sigma\tau = \tau\sigma$

$$0 = -D(\sigma(w\alpha r))\beta\theta(\sigma\tau(v))\beta G(\tau(w),\tau(r))_\alpha - D(\sigma(w\alpha u))\beta\theta(\sigma\tau(v))\beta G(\tau(r),\tau(w))_\alpha +$$

$$D(\sigma^2(w))\alpha\theta(\tau\sigma(r))\beta\theta(\sigma\tau(v))\beta\theta(\tau(w\alpha r)) + D(\sigma^2(r))\alpha\theta(\tau\sigma(w))\beta\theta(\sigma\tau(v))\beta$$

$$\theta(\tau(w\alpha r)) - D(\sigma(w))\alpha\theta(\sigma^2\tau(r))\beta\theta(\sigma^2\tau^2(v)\beta\theta(\sigma\tau^2(r))\alpha\theta(\tau(w)) -$$

$$D(\sigma(r))\alpha\theta(\sigma^2\tau(w))\beta\theta(\sigma^2\tau^2(v)\beta\theta(\sigma\tau^2(w))\alpha\theta(\tau(r)), \text{since } \sigma^2 = \sigma \text{ with } \tau^2 = \tau$$

$$0 = -D(\sigma(w\alpha r))\beta\theta(\sigma\tau(v))\beta G(\tau(w),\tau(r))_\alpha - D(\sigma(w\alpha r))\beta\theta(\sigma\tau(v))\beta G(\tau(r),\tau(w))_\alpha +$$

$$D(\sigma(w))\alpha\theta(\tau\sigma(r))\beta\theta(\sigma\tau(v))\beta\theta(\tau(w\alpha r)) + D(\sigma(r))\alpha\theta(\tau\sigma(w))\beta\theta(\sigma\tau(v))\beta$$

$$\theta(\tau(w\alpha r)) - D(\sigma(w))\alpha\theta(\tau\sigma(r))\beta\theta(\sigma\tau(v)\beta\theta(\sigma\tau(r))\alpha\theta(\tau(w)) -$$

$$D(\sigma(r))\alpha\theta(\tau\sigma(w))\beta\theta(\sigma\tau(v)\beta\theta(\sigma\tau(w))\alpha\theta(\tau(r))$$

$$0 = -D(\sigma(w\alpha r))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)G(\tau(w),\tau(r))_\alpha - D(\sigma(w\alpha u))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)G(\tau(r),\tau(w))_\alpha +$$

$$D(\sigma(w))\ k\ (\alpha)\theta(\tau\sigma(r))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)(\theta(\tau(w\alpha r)) - \theta(\sigma\tau(r))\ k\ (\alpha)\theta(\tau(w))) +$$

$$D(\sigma(r))\ k\ (\alpha)\theta(\tau\sigma(w))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)(\theta(\tau(w\alpha r)) - \theta(\sigma\tau(w))\ k\ (\alpha)\theta(\tau(r)))$$

$$0 = -D(\sigma(w\alpha r))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)G(\tau(w),\tau(r))_\alpha - D(\sigma(w\alpha r))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)G(\tau(r),\tau(w)_\alpha +$$

$$D(\sigma(w))\ k\ (\alpha)\theta(\tau\sigma(r))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)G(\tau(r),\tau(w))_\alpha +$$

$$D(\sigma(r))\ k\ (\alpha)\theta(\tau\sigma(w))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)G(\tau(w),\tau(r))_\alpha$$

$$0 = -(D(\sigma(w\alpha r) - D(\sigma(r))\ k\ (\alpha)\theta(\tau\sigma(w)))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)G(\tau(w),\tau(r))_\alpha -$$

$$(D(\sigma(w\alpha r) - D(\sigma(w))\ k\ (\alpha)\theta(\tau\sigma(r)))\ k\ (\beta)\theta(\sigma\tau(v))\ k\ (\beta)G(\tau(r),\tau(w))_\alpha$$

Thus, we have:

$$\delta(\sigma(w),\sigma(u))_\alpha\beta\theta(\sigma\tau(v))\beta G(\tau(u),\tau(w))_\alpha + \delta(\sigma(u),\sigma(w))_\alpha\beta\theta(\sigma\tau(v))\beta G(\tau(w),\tau(u))_\alpha = 0$$

(ii) Replace α by β in (i), we obtain (ii).

(iii) Interchanging α with β in (i), we obtain (iii).

2.14. Lemma

Assume D be a generalized Jordan (σ,τ)-k-homomorphism of a Γ-ring N into a two- torsion free prime Γ́ -ring Ń, then for any *w*, *r*, v ∈ N withα, β ∈ Γ

(i) $\delta(\sigma(w),\sigma(r))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(r),\tau(w))_\alpha=\delta(\sigma(r),\sigma(w))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(w),\tau(r))_\alpha=0$

(ii) $\delta(\sigma(w),\sigma(r))_\alpha$ k (α)θ(στ(v)) k (α)$G(\tau(r),\tau(w))_\alpha=\delta(\sigma(r),\sigma(w))_\alpha$ k (α)θ(στ(v)) k (α)$G(\tau(w),\tau(r))_\alpha=0$

(iii) $\delta(\sigma(w),\sigma(r))_\beta$ k (α)θ(στ(v)) k (α)$G(\tau(r),\tau(w))_\beta=\delta(\sigma(r),\sigma(w))_\beta$ k (α)θ(στ(v)) k (α)$G(\tau(w),\tau(r))_\beta=0$

Proof:

(i) By .(2.10) (i) Lemma

$\delta(\sigma(w),\sigma(r))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(r),\tau(w))_\alpha+\delta(\sigma(r),\sigma(w))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(w),\tau(r))_\alpha=0$

With since by Lemma (Assume M be a two-torsion free semi-prime Γ-ring with assume that *w, r* ∈ N if *w*ΓvΓ*r* + *r*ΓvΓ*w* = 0 for any v ∈ N, then *w*ΓvΓ*r* = *u*ΓvΓ*w* = 0), we obtain

$\delta(\sigma(w),\sigma(r))_\alpha k(\beta)\theta(\sigma\tau(v))k(\beta)G(\tau(r),\tau(w))_\alpha=\delta(\sigma(r),\sigma(w))_\alpha k(\beta)\theta(\sigma\tau(v))k(\beta)G(\tau(w),\tau(r))_\alpha=0$

(ii) Replace α for β in (i), we obtain (ii).

(iii) Interchanging α with β in (i), we obtain (iii).

2.15. Lemma

Assume D be a generalized Jordan (σ,τ)-k-homomorphism of a Γ-ring N into a prime Γ'-ring Ń, then for any *w*, *r*, *t*, *h*, v ∈ M with α, β ∈ Γ

(i) $\delta(\sigma(w),\sigma(r))_\alpha k(\beta)\theta(\sigma\tau(v))k(\beta)G(\tau(h),\tau(t))_\alpha = 0$

(ii) $\delta(\sigma(w),\sigma(r))_\alpha k(\alpha)\theta(\sigma\tau(v))k(\alpha)G(\tau(h),\tau(t))_\alpha = 0$

(iii) $\delta(\sigma(w),\sigma(r))_\alpha k(\alpha)\theta(\sigma\tau(v))k(\alpha)G(\tau(h),\tau(t))_\beta = 0$

Proof:

(i) replacing $w + t$ for a in Lemma (2.11) (i), we obtain:

$\delta(\sigma(w + t),\sigma(r))_\alpha k(\beta)\theta(\sigma\tau(v))$ $k(\beta)G(\tau(r),\tau(w + t))_\alpha = 0$

$\delta(\sigma(a),\sigma(r))_\alpha$ k (β)θ(στ(t)) k (β)$G(\tau(r),\tau(w))_\alpha+\delta(\sigma(w),\sigma(r))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(r),\tau(t))_\alpha+$

$\delta(\sigma(t),\sigma(r))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(r),\tau(w))_\alpha+\delta(\sigma(t),\sigma(r))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(r),\tau(t))_\alpha= 0$

By lemma (2.11)(i), we obtain:

$\delta(\sigma(w),\sigma(r))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(r),\tau(t))_\alpha+\delta(\sigma(t),\sigma(r))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(r),\tau(w))_\alpha= 0$

Hence, we obtain

$\delta(\sigma(w),\sigma(r))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(r),\tau(t))_\alpha$ k (β)θ(στ(v)) k (β)$\delta(\tau(w),\tau(r))_\alpha$ k (β)θ(στ(v)) k (β)

$G(\tau(r),\tau(t))_\alpha= 0$

= – $\delta(\sigma(w),\sigma(r))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(r),\tau(t))_\alpha$ k (β)θ(στ(v)) k (β)$\delta(\tau(t),\tau(r))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(r),\tau(w))_\alpha= 0$

Since Ń is prime with hence:

$\delta(\sigma(w),\sigma(r))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(r),\tau(t))_\alpha = 0$ …(1)

Now, replacing r + h for r in Lemma (2.11)(i), we obtain:

$\delta(\sigma(w),\sigma(r+h))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(r+h),\tau(w))_\alpha = 0$

$\delta(\sigma(w),\sigma(r))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(r),\tau(w))_\alpha+\delta(\sigma(w),\sigma(r))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(h),\tau(w))_\alpha+$

$\delta(\sigma(w),\sigma(h))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(r),\tau(w))_\alpha+\delta(\sigma(w),\sigma(h))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(h),\tau(w))_\alpha=0$

By lemma (2.11)(i), we obtain:

$\delta(\sigma(w),\sigma(r))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(h),\tau(w))_\alpha+\delta(\sigma(w),\sigma(h))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(r),\tau(w))_\alpha = 0$

Hence, we obtain:

$\delta(\sigma(w),\sigma(r))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(h),\tau(w))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)\delta(\sigma(w),\sigma(r))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(h),\tau(w))_\alpha = 0$

$= -\delta(\sigma(w),\sigma(r))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(h),\tau(w))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)\delta(\sigma(w),\sigma(h))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(r),\tau(w))_\alpha = 0$

Since Ń is prime with hence:

$\delta(\sigma(w),\sigma(r))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(h),\tau(w))_\alpha = 0$ …(2)

Now, $\delta(\sigma(w),\sigma(u))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(u+h),\tau(w+t))_\alpha = 0$

$\delta(\sigma(w),\sigma(r))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(r),\tau(w))_\alpha+\delta(\sigma(w),\sigma(r))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(r),\tau(t))_\alpha+$

$\delta(\sigma(w),\sigma(r))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(h),\tau(w))_\alpha+\delta(\sigma(w),\sigma(r))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(h),\tau(t))_\alpha = 0$

Since by lemma (2.11) (i) with (1), (2), we obtain:

$\delta(\sigma(w),\sigma(u))_\alpha$ k $(\beta)\theta(\sigma\tau(v))$ k $(\beta)G(\tau(h),\tau(t))_\alpha = 0$.

(ii) Replace α for β in (i), we obtain (ii).
(iii) Replace $\alpha+\beta$ for α in (ii), we obtain:
$\delta(\sigma(w),\sigma(r))_{\alpha+\beta}$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)G(\tau(h),\tau(t))_{\alpha+\beta} = 0$

$\delta(\sigma(w),\sigma(r))_\alpha$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)G(\tau(h),\tau(t))_\alpha+\delta(\sigma(w),\sigma(r))_\alpha$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)G(\tau(h),\tau(t))_\beta +$

$\delta(\sigma(w),\sigma(r))_\beta$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)G(\tau(h),\tau(t))_\alpha+\delta(\sigma(w),\sigma(r))_\beta$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)G(\tau(h),\tau(t))_\beta = 0$

By (i) with (ii), we obtain:

$\delta(\sigma(w),\sigma(u))_\alpha$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)G(\tau(h),\tau(t))_\beta+\delta(\sigma(w),\sigma(r))_\beta$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)G(\tau(h),\tau(t))_\alpha = 0$

Hence, we have:

$\delta(\sigma(w),\sigma(r))_\alpha$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)G(\tau(h),\tau(t))_\beta$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)\delta(\tau(w),\tau(r))_\alpha$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)G(\tau(h),\tau(t))_\beta = 0$

$= -\delta(\sigma(w),\sigma(r))_\alpha$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)G(\tau(h),\tau(t))_\beta$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)\delta(\tau(w),\tau(r))_\beta$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)G(\tau(h),\tau(t))_\alpha= 0$

Since Ḿ is prime , then:

$\delta(\sigma(w),\sigma(r))_\alpha$ k $(\alpha)\theta(\sigma\tau(v))$ k $(\alpha)G(\tau(h),\tau(t))_\beta = 0$.

3. Results and Discussion

3.1. Theorem

Any generalized Jordan (σ,τ)-k-homomorphism of a Γ-ring N into prime Γ́ -ring Ń is either generalized (σ,τ)-k-homomorphism or (σ,τ)-k -anti homomorphism.

3.2. Proof:

Assume D be a generalized Jordan (σ,τ)-k-homomorphism of a Γ-ring N into prime Γ́ -ring Ń , then by Lemma (2.12) (i):

$\delta(\sigma(w),\sigma(r))_\alpha$ k (β)θ(στ(v)) k (β)$G(\tau(h),\tau(t))_\alpha = 0$.

Since Ń is prime Γ́-ring hence either $\delta(\sigma(w),\sigma(r))_\alpha$=o or $G(\tau(h),\tau(t))_\alpha$=0 for any *w*, *r*, *t*, *h* ∈ N with α ∈ Γ.

If $G(\tau(h),\tau(t))_\alpha \neq$ o for any *t*, *h* ∈ N with α ∈ Γ then $\delta(\sigma(w),\sigma(r))_\alpha$ = o, hence we obtain F is generalized (σ,τ)-k-homomorphism.

But if $G(\tau(h),\tau(t))_\alpha$ = o for any *t, h* ∈ N with α ∈ Γ, then we obtain D is (σ,τ)-k -anti homomorphism.

3.3. Proposition

Assume D bɐ a generalized Jordan (σ,τ)-k-homomorphism from a Γ-ring N into a two-torsion free Γ́-ring Ń , such that *w*α*r*β*w* = *w*β*r*α*w*, for any *w*, *r* ∈ N with α, β ∈ Γ, *w*'α'*r*'β'*w*' = *w*'β'*r*'α'*w*', for any *w*', *r*' ∈ Ń with α', β'∈ Γ́, $\sigma^2 = \sigma$, $\tau^2 = \tau$ with στ = τσ, then D is a generalized Jordan triple (σ,τ)-k-homomorphism.

Proof:

Replace *r* by *w*β*r* + *r*β*w* in Definition (2.3), we obtain:

D (*w*α(*w*β*r* + *r*β*w*) + (*w*β*r* + *r*β*w*)α*w*) = D(σ(*w*)) k (α)θ(τ(*w*β*r* + *r*β*w*)) +

D (σ(*w*β*r* + *r*β*w*)) k (α)θ(τ(*w*))

= D (σ(*w*)) k (α)θ(τ(*w*)βτ(*r*) + τ(*r*)βτ(*w*)) + D(σ(*w*)βσ(*r*) + σ(*r*)βσ(*w*)) k (α)θ(τ(*w*))

= D (σ(*w*)) k (α)θ(στ(*w*)) k (β)θ(τ^2(*r*)) + D(σ(*w*)) k (α)θ(στ(*r*)) k (β)θ(τ^2(*w*)) +

D (σ^2(*w*)) k (β)θ(τσ(*r*)) k (α)θ(τ(*r*)) + D(σ^2(*u*)) k (β)θ(τσ(*w*)) k (α)θ(τ(*w*))

Since *w*'α'*r*'β'*h*' = *w*'β'*r*'α'*w*', for any *w*', *r*' ∈ Ń with α', β'∈ Γ', $\sigma^2 = \sigma$, $\tau^2 = \tau$ with στ = τσ, we obtain:

=D (σ(*w*)) k (α)θ(στ(*w*)) k (β)θ(τ(*r*))+2D(σ(*w*)) k (α)θ(στ(*r*)) k (β)θ(τ(*w*))+D(σ(*r*)) k (β)

θ(στ(*w*)) k (α)θ(τ(*w*)) ...(1)

In contrast:

D (*w*α(*w*β*r* + *r*β*w*) + (*w*β*r* + *r*β*w*)α*w*) = D(*w*α*w*β*r* + *w*α*r*β*w* + *w*β*r*α*w* + *w*β*w*α*w*)

Since *w*α*r*β*w* = *w*β*r*α*w*, for any *w*, *r* ∈ N withα, β ∈ Γ, wɐ obtain:

= D(σ(*w*)) k (α)θ(στ(*w*)) k (β)θ(τ(*r*)) + D(σ(*r*)) k (β)θ(στ(*w*)) k (α)θ(τ(*w*)) + 2D(*w*α*r*β*w*) ...(2)

Evaluate (1) with (2), we obtain:

2D (*w*α*r*β*w*) = 2D (σ(*w*)) k (α)θ(στ(r)) k (β)θ(τ(*w*)).

Since Ń is a two-torsion free Γ́-ring, we obtain D is a generalized Jordan triple (σ,τ) - k-homomorphism.

The present paper explores generalized Jordan (σ,τ)-k-homomorphism in the context of the Γ-rings a generalization of classical ring theory in which multiplication is defined by an operator set Γ. This generalization of prior Jordan homomorphism and (σ,τ)-mappings context includes a mapping $k : \Gamma \rightarrow \Gamma'$ which adds to the homomorphism structure [4-6].

The contribution of the key players is twofold:

3.4. Dichotomy Theorem:

Given the assumption that the target Γ-ring N′ is two torsion free and prime, any generalized Jordan (σ,τ)-k-homomorphism $D : N \rightarrow N'$ must be either:

A (σ,τ)-k homomorphism, or generalized (σ,τ)-k-homomorphism.

This fact illustrates the inflexibility of such mappings in prime environments, where the condition of the Jordan forces certain multiplicative condition to a certain behavior [6-8].

Jordands to Triple Homomorphism: Basically theory is a complicated perspective which uses a product designing approach to display appeared behavior, possibility, constraint, and demonstration.

Given further conditions, such as the idempotence and commutativity of σ and τ, two-torsion freeness of N and N' and some commutativity properties in N and N', every generalized Jordan (σ,τ)-k-homomorphism is also a generalized Jordan triple (σ,τ)-k-homomorphism. This will fill two significant gaps in the study of mappings and demonstrate that, in well-behaved, Γ-rings Jordan-type conditions will often define any halter triple-raising properties [5-7].

It is compatible with and generalizes classical theorems on Jordan homomorphism of Jacobson and Rickart to the Γ -ring setting of operator maps (σ, τ, k).

The prime condition is extremely important, which permits invoking annihilation lemmas to coerce structure conclusions.

It has been further extended by the addition of flexibility of the k-map to general algebraic systems, in which the sets of operators can change between domain and codomain [4-6].

The present work has provided a number of research directions, such as: Primacy laxing to semi-primacy (or other ideal theorized conditions), Research on generalized (σ,) -k derivations and mappings.

The study of Γ-rings Lie structures, orthogonality, and centrality. Two dimensions further Ternary rings, super rings, and other non-associative algebras. Overall, this paper will contribute to the structural theory of Γ-rings with the classification of generalized Jordan (σ,τ)-k-homomorphism and the disclosure of the circumstances in which they can become triple homomorphism. The findings can be used to understand more about multiplicative mappings of generalized ring systems and form a basis to add to further algebraic research [5-8].

4. Conclusions

Generalized (σ,τ)-k-homomorphism (between Γ-rings). We have systematically studied generalized Jordan (σ,τ)-k-homomorphism and discovered the following main results:

Dichotomy Theorem: Every generalization of the Jordan proved (σ,τ)-k-homomorphism of a Γ-ring N to a prime Γ 1 homomorphism N is either a generalized (σ,τ)-k-homomorphism or a (σ,τ)-k-anti homomorphism. The absence of this behavior figures out the scanty algebraic conduct of such mappings, on a natural ring-theoretic scheme of constraints.

It was found that lifting to Triple Homomorphism with other conditions, such as two-torsion freeness, idemp Potency and commutativity of σ and τ and some conditions related to commutativity in N and N ed. N, each generalized Jordan (σ,τ)-k-homomorphism is also a generalized Jordan triple (σ,τ)-k-homomorphism. This enhances the association between Jordan and the multiplicative structures of the Gamma-rings.

Technical Lemmas and Tools: We have come up with some important lemmas (2.6, 2.8, 2.10, 2.12) that help us analyse the 2.10 2.12: 2.10-2.11: and the properties of 2.10. Such tools will probably service subsequent researches on generalized mappings in Gamma rings and within the framework of other similar structures.

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