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Exponential-gamma-Rayleigh distribution and its applications

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Abstract

probability distribution help researcher and practitioners understand and model complex behaviour of rainfall data, ultimately behaviour of rainfall data and decision making in field of hydrology, water resource management and climate change impact assessment which intensify for specific duration simulation event and generate synthetic rainfall data and also optimize water resource management by modelling the probability of future rainfall scenarios Understanding and interpreting data behaviour more scientifically is an essential stage in every field of life. Statistical methods are used in applied in fields of hydrological, and mesosphere and lower thermosphere weather observations. Several researchers have generated new adaptable distributions from existing distributions using various modification techniques to increase their flexibility in rainfall modelling data. These adaptable distributions are created by adding extra parameters to the baseline distribution with generators or combining two distributions (Ali, et al., 2021). These modified distributions can model data sets efficiently and in most case, provide the best fit to data sets when applied because they have more parameters and are more adaptable than their baseline distributions. Data on the thirty observations for March rainfall in Minneapolis/St Paul (in inches), the data set has been used by Isa, et al., (2022), data sets obtained from Lee and Wang, (2003), and , the data set obtained from Fatima and Ahmad, (2017), which represents the 72 exceedances of flood maxima (in m3/s) of the Wheaton River near Carcoss in Yukon Territory, Canada, from 1958 to 1984 (rounded to one decimal point). The newly developed probability distributions robustness and versatility are evaluated by comparing them to other related existing probability distributions, such as the Exponential, Gamma, and Rayleigh distributions. Also, the Exponential-Gamma distribution developed by Ogunwale, et al., (2019), using goodness of fit measurements The Python 3.10.10 software package was used to analyse the data. The Akaike information criterion (AIC), Bayesian information criterion (BIC), and log-likelihood function (l) are the goodness of fit measures discussed. The probability distribution with the lowest Akaike information criterion (AIC), Bayesian information criterion (BIC), or highest log-likelihood function (l) value will be used to determine the best-suited model.

Keywords: Exponetial-Gamma-Rayleigh; Cumulative density function Maximum likelihood extimator Alkaike information Criterion; Bayesian information Criterion; Model

1. Introduction

The use of traditional probability models to forecast real-world events is causing growing dissatisfaction among scholars. Many applications, such as lifetime analysis require extended forms of these distributions. Most statistical distribution modelling approaches are concerned with determining which probability distribution best represents the data. The results of fitting these data sets with traditional distributions may be unreliable (Klakatawi, *et al*., 2022). No single probability distribution, however, can fit all types of data. As a result, new classical distributions must be developed or created (Nasiru, 2018). One of the motives could be that the tail characteristics and goodness of fit metrics

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have a constraining tendency. Subsequently, there has been a significant increase in the generalisation of well-known probability distributions in recent years. The challenge is to find families that are versatile enough to fit both skewed and symmetric data. It is essential to understand that most generalised distributions described in the literature were developed using the generalised transformed transformer (T-X) method. This method was proposed by Alzaatreh, *et al*., (2013), also Adewusi, *et al*., (2019) show that this generalisation approach is beneficial by transforming the Exponential-Gamma distribution developed by Ogunwale, *et al*., (2019), to a family of distribution known as the Exponential-Gamma-X. Hence, in this study, we explore the T-X approach by adding more parameters to the Exponetial-Gamma-X distribution, thereby providing a more flexible probability distribution. The solutions to the enormous majority of problems in probability theory are sometimes obtained by applying limit theorems, however these solutions are only approximate., the lack of an explicit formula for the remainder terms makes estimating errors problematic considering this fact, it is critical to concentrate on methods that can yield more precise solutions and functional transformation of random variables is the fundamental basis of such methods, while in practice, we may encounter data with monotonic or non-monotonic hazard rate shapes in some cases, such as life distributions available models may fail to apply. As a result, researchers are attempting to develop models that account for the limitations of these distributions. So, in this study, we develop a model that is efficiently fit such data. This purpose is achieved by adding an extra parameter using generators or by the existing models, therefore, in this study, we develop a new probability distribution called the Exponential-Gamma-Rayleigh distribution by using the pdf of the new Exponential-Gamma-X distributions developed by Adewusi, *et al*., (2019) and apply it to rainfall data.

2. Methods

The Exponential-Gamma Rayleigh distribution was developed by Adisa et al in 2025 and the pdf is defined as

$$g(x)=\frac{\theta^{\alpha}2\lambda}{\Gamma(\alpha)}x^{2\alpha-1}\exp\left(-2\theta x\right),\ x,\lambda,\alpha,\theta>0 \quad \text{………….} \qquad (1)$$

With the mean and variance

$$\mu_r^{'}=\frac{2\theta^{\alpha}\lambda\Gamma(2\alpha+r)}{\left(2\theta\right)^{2\alpha+r}\Gamma(\alpha)} \quad \text{………….} \ (2)$$

And

The variance, σ^2, is given as $V(x)=\mu_2'-\left(\mu_1'\right)^2$

$$V(x)=\frac{2\alpha\lambda(2\alpha+1)\Gamma(2\alpha)}{2^{2\alpha+1}\theta^{\alpha+2}\Gamma(\alpha)}-\left(\frac{2\alpha\lambda\Gamma(2\alpha)}{2^{2\alpha}\theta^{\alpha+1}\Gamma(\alpha)}\right)^2 \quad \text{………….} \qquad (3)$$

$$V(x)=\frac{\alpha\lambda(2\alpha+1)2^{2\alpha}\theta^{\alpha}-4\alpha^2\lambda^2}{2^{4\alpha}\theta^{2\alpha+2}} \quad \text{………….} \ (4)$$

The Cumulative distribution function is define as

$$F(x)=\frac{2\lambda\gamma(2\alpha,x)}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)}\quad x,\theta,\alpha,\lambda>0 \quad \text{………….} \quad (5)$$

CDF

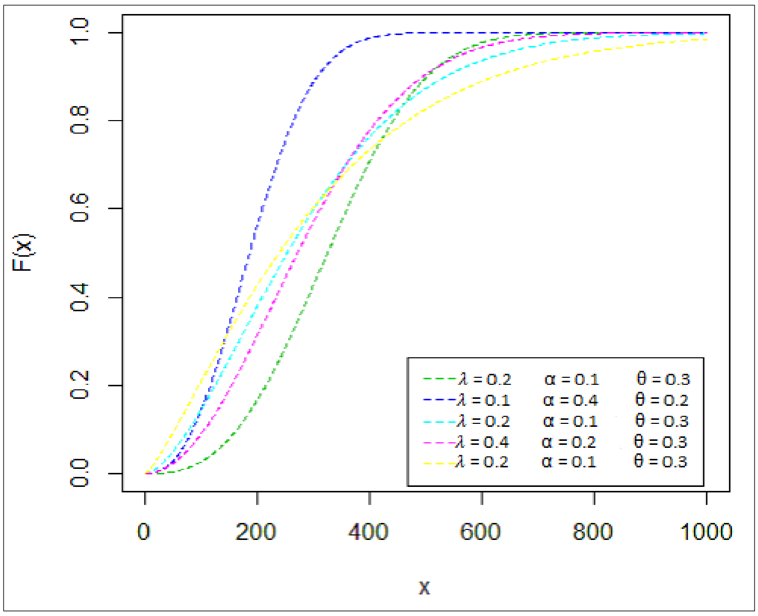

Figure 1 CDF plots for EGRD

The cumulative distribution function (CDF) plot of the EGRD starts from zero on the y axis and tend to 1 on x axis by assigning different values to the parameters, which is an indication that the EGRD is a valid distribution because it satisfies the basic property of a valid probability distribution which states that the probability of any event is greater than or equal to zero and the sum of the cumulative probabilities of events is equal to one.

The survival function function is define as

$$S(x) = 1 - F(x) \quad .... (6)$$

where $F(x)$ is the cumulative distribution function of *X*, substituting (6) in (5) then,

$$S(x) = \frac{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha) - 2\lambda\gamma(2\alpha, x)}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)} \quad .... (7)$$

While the corresponding hazard function function is define by

$$S(x) = 1 - \frac{2\lambda\gamma(2\alpha, x)}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)} \quad .... (8)$$

$$h(x) = \frac{f(x)}{S(x)} \quad .... (9)$$

where $f(x)$ and $S(x)$ are pdf and survival function of EGRD then,

$$h(x) = \frac{\dfrac{2\lambda\theta^{\alpha}x^{2\alpha-1}e^{-2\theta.x}}{\Gamma(\alpha)}}{\dfrac{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha) - 2\lambda\gamma(2\alpha, x)}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)}} \quad .... (10)$$

$$= \frac{2\lambda\theta^{\alpha} x^{2\alpha-1} e^{-2\theta.x}}{\Gamma(\alpha)} \text{ x } \frac{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)-2\lambda\gamma(2\alpha,x)} \quad \ldots\ldots\ldots\ldots\ldots (11)$$

$$h(x) = \frac{2^{2\alpha+1}\theta^{2\alpha}\lambda x^{2\alpha-1}e^{-2\theta.x}}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)-2\lambda\gamma(2\alpha,x)} \quad \ldots\ldots\ldots\ldots\ldots (12)$$

The cumulative hazard function for distribution is define by

$$H(x) = W(F(x)) = -\log(1-F(x)) \equiv \int_0^x h(x)dx \quad \ldots\ldots\ldots\ldots\ldots (13)$$

$$H(x) = \int_0^x \frac{2^{2\alpha+1}\lambda\theta^{2\alpha}x^{2\alpha-1}e^{-2\theta.x}}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)-2\lambda\gamma(2\alpha,x)}dx \quad \ldots\ldots\ldots\ldots\ldots (14)$$

$$= \frac{2^{2\alpha+1}\lambda\theta^{2\alpha}}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)-2\lambda\gamma(2\alpha,x)}\int_0^x x^{2\alpha-1}x^{-2\theta.x}dx \quad \ldots\ldots\ldots\ldots\ldots (15)$$

let $u = 2\theta x,\ \ x = \frac{u}{2\theta}$, then $dx = \frac{du}{2\theta}$ so that (15) is reduced to:

$$= \frac{2^{2\alpha+1}\lambda\theta^{2\alpha}}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)-2\lambda\gamma(2\alpha,x)}\cdot\frac{1}{(2\theta)^{2\alpha}}\int_0^x u^{2\alpha-1}e^{-u}du \quad \ldots\ldots\ldots\ldots\ldots 16)$$

$$= \frac{2^{2\alpha+1}\lambda\theta^{2\alpha}}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)-2\lambda\gamma(2\alpha,x)}\cdot\frac{1}{(2\theta)^{2\alpha}}\int_0^x u^{2\alpha-1}e^{-u}du \quad \ldots\ldots\ldots\ldots\ldots (17)$$

recall that $\int_0^x u^{2\alpha-1}e^{-u}du = \gamma(2\alpha,x)$ is an incomplete gamma function then,

$$= \frac{2^{2\alpha+1}\lambda\theta^{2\alpha}\gamma(2\alpha,x)}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)-2\lambda\gamma(2\alpha,x)}\cdot\frac{1}{(2\theta)^{2\alpha}} \quad \ldots\ldots\ldots\ldots\ldots (18)$$

$$H(x) = \frac{2\lambda\gamma(2\alpha,x)}{2^{2\alpha}\theta^{\alpha}\Gamma(\alpha)-2\lambda\gamma(2\alpha,x)}. \quad \ldots\ldots\ldots\ldots\ldots (19)$$

3. A maximum Likelihood Estimator

Let $x_1, x_2, \ldots\ldots x_n$ be a random sample of size n from Exponential-Gamma-Rayleigh distribution (EGRD) with pdf

$$\mathrm{L}(\alpha,\lambda,\beta;x) = \left(\frac{2\lambda\theta^{\alpha}}{\Gamma(\alpha)}\right)^n \prod_{i=1}^n x_i^{2\alpha-1}\exp\left(-2\theta\sum x_i\right) \ldots\ldots\ldots\ldots\ldots (20)$$

By taking the natural logarithm of (20), the log-likelihood function is obtained as;

$$\log_e(L) = \alpha n \log_e \theta + n \log_e 2\lambda - n \log_e \Gamma(\alpha) + (2\alpha - 1)\sum \log_e x_i - 2\theta \sum x_i \quad \text{...............} (21)$$

Differentiating equation (21) with respect to α , λ and θ give the maximum likelihood.

Therefore, the MLE which maximizes (21) must satisfy the following normal equations;

$$\frac{\partial \log_e L}{\partial \alpha} = n \log_e \theta - \frac{n\Gamma^{'}(\alpha)}{\Gamma(\alpha)} + 2\sum_{i=1}^{n} \log_e x_i = 0 \quad \text{...............} (22)$$

$$\frac{\partial \log_e L}{\partial \lambda} = \frac{n}{\lambda} = 0 \quad \text{...............} \quad (23)$$

$$\frac{\partial \log_e L}{\partial \theta} = \frac{\alpha n}{\theta} - 2\sum x_i = 0 \quad \text{...............} \quad (24)$$

Differentiating equation (22) with respect to α , λ and θ give the maximum likelihood estimates of the model parameters that generate the solution of the nonlinear system of equations. The parameters can be estimated numerically by solving (22), (23), and (24), while solving it analytically is very cumbersome and tasking. The numerical solution can also be obtained directly using some data sets in Python but there are other programming language that could do the work, while the Akaike information criterion (AIC), Bayesian information criterion (BIC), and log-likelihood function (*l*) are the goodness of fit measures used. The probability distribution with the lowest Akaike information criterion (AIC), Bayesian information criterion (BIC) or highest log-likelihood function (*l*) value is considered as the best-suited model.

3.1. Data Set 1: Rainfall Data Set

The data set was retrieved from Isa, *et al*., (2022). It includes thirty observations for March rainfall (in inches) in Minneapolis/St Paul. The data is presented below:

0.77, 1.74, 0.81, 1.20, 1.95, 1.20, 0.47, 1.43, 3.37, 2.20, 3.00, 3.09, 1.51, 2.10, 0.52, 1.62, 1.31, 0.32, 0.59, 0.81, 2.81, 1.87, 1.18, 1.35, 4.75, 2.48, 0.96, 1.89, 0.90, 2.05

3.2. Data Set 1 : The Wheaton River's Flood Peak

The data set was obtained from Fatima and Ahmad (2017) and represents the 72 exceedances of flood maxima (in m3/s) of the Wheaton River near Carcoss in Yukon Territory, Canada, from 1958 to 1984 (rounded to one decimal point). The details are as follows:

"1.7,-2.2,-14.4,-1.1,-0.4, 20.6, 5.3, 0.7, 1.9, 13.0, 12.0, 9.3, 1.4, 18.7, 8.5, 25.5, 11.6, 14.1, 22.1, 1.1, 2.5, 14.4, 1.7, 37.6, 0.6, 2.2, 39.0, 0.3, 15.0, 11.0, 7.3, 22.9, 1.7, 0.1, 1.1, 0.6, 9.0, 1.7, 7.0, 20.1, 0.4, 2.8 ,14.1, 9.9, 10.4, 10.7, 30.0, 3.6, 5.6, 30.8, 13.3, 4.2, 25.5, 3.4, 11.9, 21.5, 27.6, 36.4, 2.7, 64.0, 1.5, 2.5, 27.4, 1.0, 27.1, 20.2, 16.8, 5.3, 9.7, 27.5, 2.5, 7.0

Table 1 Summary of data (Rainfall)

Parameters	Values
N	30
Min	0.32
Max	4.75
Mean	0.915
Skewness	1.8741
Kurtosis	2.391664

Source: rainfall , Isa (2022)

Table 1 show that the data distribution is skewed to the right, with a skewness of 1.8741. This indicates that the Exponential-Gamma-Rayleigh distribution can fit data that is rightly skewed. Furthermore, the kurtosis value is 2.391664, which is less than 3 compared to the Normal distribution; this shows that the distribution has shorter and lighter tails with a light peakedness

Table 2 Estimates and performance of the distributions (Rainfall)

Distribution	Parameter Estimate	AIC	BIC	Log-likelihood(l)
EGRD	$\hat{\boldsymbol{\alpha}} = 1.9874$ $\hat{\boldsymbol{\lambda}} = 4.5487$ $\hat{\boldsymbol{\theta}} = 0.5487$	315.4587	305.1457	173.6456
EGD	$\hat{\alpha} = 1.2047$ $\hat{\lambda} = 3.1021$	325.1587	383.5487	108.6943
GD	$\hat{\alpha} = 1.9745$ $\hat{\lambda} = 1.3418$	854.8923	839.4914	93.94963
ED	$\hat{\lambda} = 1.9514$	594.7471	597.3051	25.2547
RD	$\hat{\alpha}$ =0.7406	489.6751	494.8851	61.1478

The estimates of the parameters, Akaike information criterion (AIC), Bayesian information criterion (BIC), and log-likelihood, for the data set on the thirty observations for March rainfall (in inches) in Minneapolis/St Paul are presented in Table 2. From the table; the Exponential-Gamma Reyleigh Distribution provides a better fit as compared to the Exponential, Gamma and Rayleigh distributions. Likewise, the Exponential-Gamma-Rayleigh yields the overall performance when compared to other four distributions since it has the lowest value of the Akaike information criterion (AIC) and Bayesian information criterion (BIC) and the highest value of log-likelihood (l). Hence, the Exponential-Gamma-Rayleigh distribution performed better than other distributions compared.

Table 3 Summary of data (Wheaton River's Flood Peak)

Parameters	Values
N	72
Min	0.1
Max	64
Mean	11.936
Skewness	1.2154
Kurtosis	7.8141

Source: Exceedance flood, Fatima & Ahmad (2017)

The results from Table 3 indicated that the data distribution is skewed to the right with skewness of 1.2154. This shows that the Exponential-Gamma-Rayleigh distribution can fit right-skewed data. Also, the kurtosis value is 7.8141, which is greater than 3. This implies that the distribution has longer and fatter tails with a heavy peakedness compared to the Normal distribution.

Table 4 Estimates and performance of the distributions (Wheaton River's Flood Peak))

Distribution	Parameter Estimate	AIC	BIC	Log-likelihood(l)
EGRD	$\hat{\alpha} = 1.7158$ $\hat{\lambda} = 8.2817$ $\hat{\theta} = 0.2147$	121.6175	125.3033	471.4112
EGD	$\hat{\alpha} = 1.2471$ $\hat{\lambda} = 3.2581$	285.8199	283.1941	213.9463
GD	$\hat{\alpha} = 1.5784$ $\hat{\lambda} = 3.4178$	845.8913	843.2141	83.9163
ED	$\hat{\lambda} = 1.8147$	594.7147	597.3125	96.3514
RD	$\hat{\sigma}$ =0.8745	689.6715	694.8185	42.8138

The results from Table 4shows the estimates of the parameters, Akaike information criterion (AIC), Bayesian information criterion (BIC), and log-likelihood, for the data set on 100 observations on the breaking strength of carbon fibres. The results showed that the Exponential-Gamma Distribution provides a better fit than the Exponential, Gamma and Rayleigh distributions. Likewise, the Exponential-Gamma-Rayleigh yields the overall performance since it has the lowest value of the Akaike information criterion (AIC) and Bayesian information criterion (BIC) and the highest value of log-likelihood (l). Hence, the Exponential-Gamma-Rayleigh distribution performed better than other distributions compared.

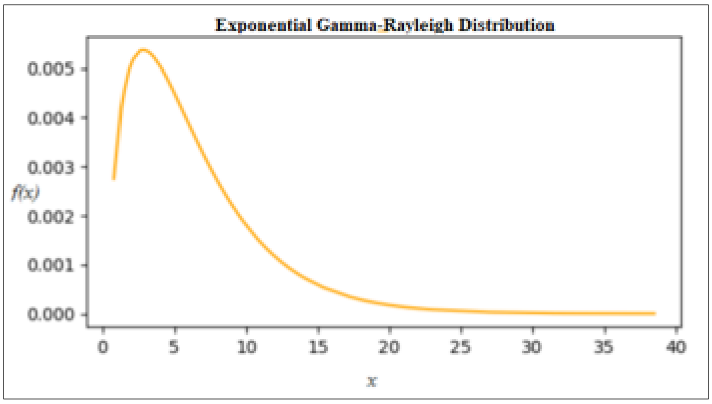

Figure 2 Exponential Gamma-Rayleigh distribution pdf plot for the data sets

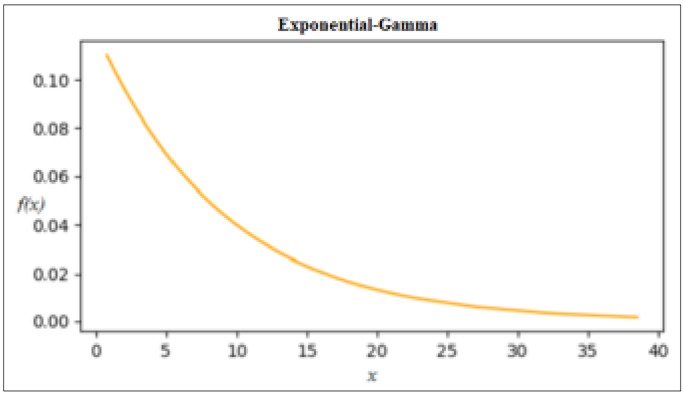

Figure 3 Exponential- gamma distribution pdf plot for the data sets

4. Conclusion

Comprehending the suitable statistical distribution to use when modeling lifetime data is critical in various fields of study. Most statistical outcomes in these fields rely heavily on distribution assumptions. On the other hand, traditionally existing distributions may be unable to appropriately reflect the data sets of some of these specialities. As a result, researchers are developing numerous generators to improve the flexibility and tractability of traditionally existing statistical distributions to give a suitable and appropriate fit to the data being analyzed.In this study well apply Exponential-Gamma-Rayleigh in modelling rainfall data and flood data. The result of the performance and adequacy of the exponential- Gamma-Rayleigh distribution was compared with other existing statistical distribution, the result showed that Exponential -Gamma Rayleigh is adequate and fit data better than the existing distribution compared for higher precision in analysing rainfall data, the use of Exponential -Gamma -Raleigh is highly recommend in various field where analysis of hydrology data is crucial.

Compliance with ethical standards

The newly developed probability distribution proves more robust, tractable and capable than those considered. It can also fit the data of different kinds better than the other distributions studied. As a result, in many fields where real-life data analysis is essential, the application of the Exponential-Gamma-Rayleigh distribution is highly recommended.

Disclosure of conflict of interest

No conflict of interest to be disclosed.

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