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Mathematical Modelling of Insecurity in Northern Nigeria: The Roles of Elites, Technology and Constitutional Bias

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Abstract

This paper constructs and examines a deterministic compartmental dynamical framework to capture insecurity dynamics in Northern Nigeria arising from elite influence, technological facilitation, and constitutional bias. The population is stratified into susceptible individuals, insecure individuals, active violent elites, exposed individuals, technologically enabled actors, and recovered individuals. Fundamental qualitative properties of the model, including positivity, invariant region and boundedness, are established. The insecurity-free equilibrium is derived and its local stability analyzed using the next-generation matrix approach, leading to the formulation of a control reproduction number R_c. Global stability of the insecurity free and endemic equilibria was also carried out. Sensitivity analysis reveals that insecurity transmission and constitutional bias are the most influential parameters sustaining instability, while recovery, institutional reform, and technological disruption significantly reduce persistence. Numerical simulations confirm the global asymptotic stability of the endemic equilibrium when $R_c>1$, demonstrating the structural persistence of insecurity. The model provides quantitative insights into how governance reforms and social interventions can effectively mitigate insecurity in Northern Nigeria.

Keywords: Constitutional bias; Control Reproduction Number; Global Stability; Insecurity Dynamics; Northern Nigeria

1 Introduction

Insecurity continues to pose a long-term structural challenge in fragile and transitional states, with Northern Nigeria experiencing recurring insurgency, banditry, and communal violence despite repeated security operations. Earlier studies largely associate insecurity with socio-economic deprivation, unemployment, religious radicalization, and limited state capacity; more recent work, however, highlights deeper structural and political mechanisms that sustain violent outcomes [1],[7],[5].

Insights from elite theory and political economy indicate that insecurity may persist endogenously when political and economic elites derive strategic or material advantages from prolonged instability [9],[6],[8]. Concurrently, advances in digital communication and encrypted platforms have expanded the recruitment reach, coordination efficiency, and operational durability of violent group [4],[2]. Furthermore, constitutional and institutional frameworks, especially those marked by centralized security authority and exclusionary governance practices, have been shown to constrain accountability and weaken local response mechanisms [10],[5].

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Notwithstanding these contributions, most existing studies remain qualitative or econometric in nature, with minimal use of modelling structure capable of capturing the dynamic interaction of political, technological, and institutional factors [3],[11]. A synthesis of the existing literature reveals several unresolved gaps, including the absence of integrated dynamic models that jointly account for elite incentives, technological amplification, and constitutional bias within insecurity systems

Therefore, motivated by these gaps, we proposes a novel compartmental mathematical model that explicitly captures elite behaviour as a dynamic component of insecurity, models technology as an amplifier of violence and coordination, quantifies constitutional bias as a structural modifier of insecurity transmission, and finally provides analytical thresholds for insecurity persistence and elimination. This model is adaptable to Northern Nigeria and other fragile contexts by integrating political, technological, and institutional factors into a unified dynamical system. These model advances both the theoretical and applied literature on insecurity.

2 Model Formulation and Assumptions

2.1 Insecurity model formulation

We formulate a deterministic compartmental dynamical system that captures insecurity propagation under elite influence, technological amplification, and institutional bias, with particular relevance to Northern Nigeria. The model extends classical social contagion frameworks by embedding political economy and institutional factors into the transmission structure. The total population $N(t)$ is partitioned into six distinct and non-overlapping compartments six namely; Susceptible individuals $S(t)$, Insecure or radicalized individuals $I(t)$, Active violent groups $A(t)$, Elite actors $E(t)$, Technological facilitators $T(t)$, and Recovered or reintegrated individuals $R(t)$. Susceptible individuals are introduced into the system at the rate Λ and become insecure through interaction with insecure individuals and violent groups at the rate of β, amplified by constitutional bias k. Insecure individuals progress to active violent groups under elite influence α. Violent groups grow through recruitment, technological reinforcement τ, and external inflows σ. Elite actors expand as violence persists, reinforcing incentives for instability. Technology evolves in response to violent activity, further amplifying insecurity. Recovery and reform reduce the insecure and violent populations at the rate of γ and δ.

2.2 Model Assumptions

The model development rests on the following simplifying premises:

- Insecurity spreads through social interaction and exposure, analogous to a contagion process.
- Elite actors have incentives to sustain insecurity and influence recruitment into violent groups.
- Technology amplifies the effectiveness, coordination, and persistence of violent actors.
- Constitutional bias weakens institutional response and increases susceptibility to insecurity.
- Recovery and reform processes reduce insecurity but operate under institutional constraints.
- All demographic and transition parameters are non-negative and constant over the study period.

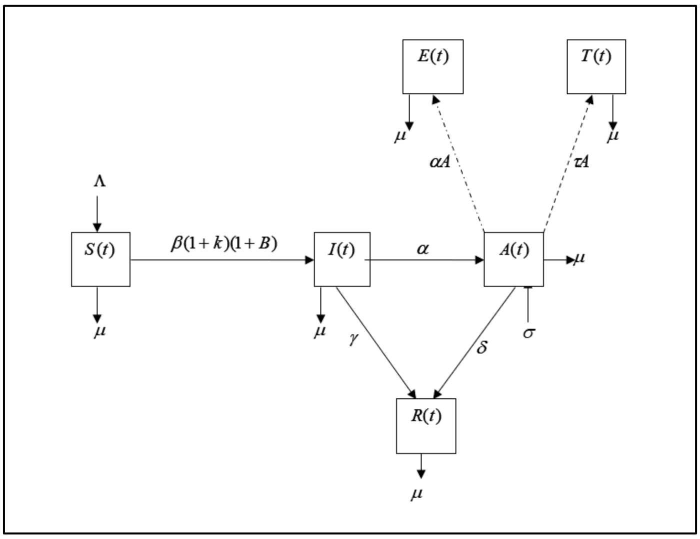

Figure 1 Insecurity model flow diagram

3 Mathematical Analysis of the Model

3.1 Insecurity equations of the model

Drawing from the preceding insecurity model flow diagram and associated assumptions, the dynamics of insecurity are described by the following system of ordinary differential equations;

$$\frac{dS}{dt} = \Lambda - \beta(1+k)(I+A)S - \mu S\,, \tag{1}$$

$$\frac{dI}{dt} = \beta(1+k)(I+A)S - (\alpha+\gamma+\mu)I\,, \tag{2}$$

$$\frac{dA}{dt} = \alpha I + \sigma - (\tau+\alpha+\delta+\mu)A\,, \tag{3}$$

$$\frac{dE}{dt} = \alpha A - \mu E\,, \tag{4}$$

$$\frac{dT}{dt} = \tau A - \mu T\,, \tag{5}$$

$$\frac{dR}{dt} = \gamma I + \delta A - \mu R\,. \tag{6}$$

Table 1 Population compartments of the insecurity model

Variables	Descriptions
S	Susceptible population who are not actively involved in violence yet are exposed to insecurity through social, economic, and political interactions.
I	Insecure or radicalized individuals who are vulnerable to recruitment or indirectly participate in insecurity-related activities.
A	Active violent groups, including insurgents, bandits, militias, and organised criminal actors.
E	Elite actors who benefit politically or economically from persistent insecurity and may indirectly sustain violent dynamics.
T	Technological facilitators, representing the level of access to communication technologies, arms logistics, and digital platforms that enhance coordination and operational efficiency of violent actors.
R)	Recovered or reintegrated individuals who have exited insecurity through rehabilitation, disengagement, or effective institutional interventions.

Table 2 Insecurity Model Parameters

Parameters	Descriptions
Λ	Recruitment rate into the susceptible population (births or migration).
β	Baseline insecurity transmission rate.

k	Constitutional bias index, capturing institutional inefficiencies and exclusion.
α	Elite influence rate on insecurity dynamics.
τ	Technological amplification rate of violent activities.
γ	Recovery or reintegration rate of insecure individuals.
δ	Effectiveness of institutional reform and security interventions.
μ	Natural exit rate (death or migration).
σ	External inflow rate of violent capacity (e.g., arms inflow).

3.2 Positivity of the Insecurity Model

Theorem 1:

From the model equations (1) to (6), all the parameters are assumed to be non-negative, with initial conditions which satisfy $S(0), I(0), A(0), E(0), T(0), R(0) \geq 0$ .The model (1) to (6) is said to be positive if for all t > 0, $S(t), I(t), A(t), E(t), T(t), R(t) \geq 0$.

Proof:

We will check the sign of each equation when the corresponding variable is zero.

From equation (1),

$$\text{At } S = 0, \ \frac{dS}{dt} = \Lambda \geq 0 .$$

Hence, S(t) cannot become negative.

From equation (2),

$$\text{At } I = 0, \ \frac{dI}{dt} = \beta(1+k)(I + A)S \geq 0 .$$

$$\text{Thus, } I(t) \geq 0 \text{ for all } t \geq 0 .$$

From equation (3),

$$\text{At } A = 0, \ \frac{dA}{dt} = \alpha I + \sigma \geq 0 .$$

Therefore, A(t) remains non-negative.

From equation (4),

$$\text{At } E = 0, \ \frac{dE}{dt} = \alpha A \geq 0 .$$

Hence, elite influence does not become negative.

From equation (5),

$$\text{At T = 0, } \frac{dT}{dt} = \tau A \geq 0 .$$

Then, $T(t) \geq 0$.

From equation (6),

$$\text{At R = 0, } \frac{dR}{dt} = \gamma I + \delta A \geq 0 .$$

So, R(t) remains non-negative.

Each equation satisfies the following lower bounds;

$$\frac{dS}{dt} \geq -\mu S , \quad (7)$$

$$\frac{dI}{dt} \geq -(\alpha + \gamma + \mu)I , \quad (8)$$

$$\frac{dA}{dt} \geq -(\tau + \alpha + \delta + \mu)A , \quad (9)$$

$$\frac{dE}{dt} \geq -\mu E , \quad (10)$$

$$\frac{dT}{dt} \geq -\mu T , \quad (11)$$

$$\frac{dR}{dt} \geq -\mu R \quad (12)$$

Solving these inequalities equations (7) to (12) by method of integrating factor yields;

$$S(t) \geq S(0)e^{-ut} ,$$

$$I(t) \geq I(0)e^{-(\alpha+\gamma+\mu)t} ,$$

$$A(t) \geq A(0)e^{-(\tau+\alpha+\delta+\mu)t} ,$$

$$E(t) \geq E(0)e^{-ut} ,$$

$$T(t) \geq T(0)e^{-ut} ,$$

$$R(t) \geq R(0)e^{-ut} .$$

Hence, all compartments remain non-negative for all t ≥ 0. The positivity analysis confirms the mathematical well-posedness of the proposed insecurity model, so that each state variable continues to have a meaningful interpretation as time progresses.

3.3 Invariant Region and Boundedness

This section establishes that the model admits no solutions that grow without bound and evolve within a positively invariant region of the state space. This guarantees the global well-posedness of the model and the biological-socio-political feasibility of its dynamics.

The total population N(t) is define as

$$N(t) = S + I + A + E + T + R \qquad (13)$$

Differentiating (13) with respect to t, we have,

(14)

Substituting equation (1) to (6) into (14) and simplifying further gives;

$$\frac{dN}{dt} = \Lambda + \sigma - \mu N - \tau A - \alpha A.$$

Since $A(t) \geq 0$, we have,

$$\frac{dN}{dt} \leq \Lambda + \sigma - \mu N \ .$$

Considering the comparison equation;

$$\frac{dX}{dt} \leq \Lambda + \sigma - \mu X \ , \ X(0) = N(0).$$

Solving by comparison theorem,

$$X(t) = \frac{\Lambda + \sigma}{\mu} + \left(N(0) - \frac{\Lambda + \sigma}{\mu} \right) e^{-\mu t}.$$

Thus,

$$0 \leq N(t) \leq \frac{\Lambda + \sigma}{\mu} \text{ for all } t \geq 0.$$

Since $0 \leq S, I, A, E, T, R \leq N(t)$, it follows directly that

$$0 \leq S(t), I(t), A(t), E(t), T(t), R(t) \leq \frac{\Lambda + \sigma}{\mu} \text{ for all } t \geq 0.$$

Hence, all solutions are uniformly bounded with feasible invariant region

$\Omega = \left\{ S(t), I(t), A(t), E(t), T(t), R(t) \in \Re_{+}^{6} : N(t) \leq \frac{\Lambda + \sigma}{\mu} \right\}$. The bound $\frac{\Lambda + \sigma}{\mu}$ shows that external inflows of violence capacity (σ) raise the long-term ceiling of insecurity. Strong institutional exits (μ) reduce the feasible region, shrinking insecurity dynamics.

This result ensures that the model dynamics remain mathematically consistent and confined to a feasible socio-political state space.

3.4 Insecurity-Free Equilibrium (IFE)

The insecurity-free equilibrium corresponds to a situation in which the system operates without any manifestation of insecurity or violence. At the insecurity-free equilibrium, there is no insecure individuals, no active violent groups, no elite rent extraction from violence, and no technological amplification of violence.

Therefore, $I_0 = A_0 = E_0 = T_0 = 0$. (15)

Let E_0 denote the insecurity-free equilibrium.

Setting all derivatives (1) to (6) to zero and substituting (15), we have,

From (1); $0 = \Lambda - \mu S_0 \Rightarrow S_0 = \frac{\Lambda}{\mu}$. From (6); $0 = \gamma(0) + \delta(0) - \mu R_0 \Rightarrow R_0 = 0$

From equation (3); To ensure true insecurity-free equilibrium, we must have:

$$\sigma = 0.$$

This condition implies no external inflow of violent capacity.

Then, $A_0 = 0$.

Consequently, the model admits an insecurity-free equilibrium (IFE) expressed as,

$$E_0 = \left(\frac{\Lambda}{\mu}, 0,0,0,0,0\right) \text{.with } \sigma = 0.$$

Hence, the existence of an insecurity-free equilibrium requires the elimination of external violent inflows. This underscores the importance of border control, arms regulation, and regional cooperation in Northern Nigeria.

3.5 Control Reproduction Number

This quantity captures the expected number of new insecurity occurrences triggered by one insecure individual in a population initially free of insecurity. The compartments that **generate new insecurity** are: $x = (I, T)^T$

Decomposition into F and V, by employing the next-generation matrix approach (van den Driessche & Watmough, 2002);

The terms associated with the emergence of new insecurity are defined as

$$F_1 = \beta(1+k)(I + A)S,$$

$$F_2 = 0.$$

At the IFE, $S_0 = \frac{\Lambda}{\mu}$, hence

$$F = \begin{pmatrix} \beta(1+k)\frac{\Lambda}{\mu} & \beta(1+k)\frac{\Lambda}{\mu} \\ 0 & 0 \end{pmatrix}.$$

Transition Terms (V) is given by,

$$V_1 = (\alpha + \gamma + \mu)I,$$

$$V_2 = (\tau + \alpha + \delta + \mu)A - \alpha I .$$

Hence,

$$V = \begin{pmatrix} \alpha + \gamma + \mu & 0 \\ -\alpha & \tau + \alpha + \delta + \mu \end{pmatrix},$$

The inverse of V is obtained after further solving and is given by,

$$V^{-1} = \frac{1}{(\alpha + \gamma + \mu)(\tau + \alpha + \delta + \mu)} \begin{pmatrix} \tau + \alpha + \delta + \mu & 0 \\ \alpha & \alpha + \gamma + \mu \end{pmatrix} .$$

Multiplying F and inverse of V will give;

$$FV^{-1} = \begin{bmatrix} \frac{\beta(1+k)\Lambda}{\mu}\left(\frac{1}{(\alpha + \gamma + \mu)} + \frac{\alpha}{(\alpha + \gamma + \mu)(\tau + \alpha + \delta + \mu)}\right) & \frac{\beta(1+k)\Lambda}{\mu(\tau + \alpha + \delta + \mu)} \\ 0 & 0 \end{bmatrix}$$

The control reproduction number R_c is determined by the principal eigenvalue of FV^{-1} . Therefore,

$$R_c = \frac{\beta(1+k)\Lambda}{\mu(\alpha + \gamma + \mu)}\left(1 + \frac{\alpha}{\tau + \alpha + \delta + \mu}\right) \qquad (15a)$$

The control reproduction number provides a sharp threshold that links elite behaviour, technological amplification, and constitutional bias to the persistence or elimination of insecurity.

3.6 Local Stability of the Insecurity-Free Equilibrium

Theorem 2:

The insecurity-free equilibrium $E_0$is locally asymptotically stable if $R_c< 1$, and unstable if $R_c> 1$.

Proof:

The Jacobian matrix evaluated at E_0 has eigen-values: $-\mu$ (three times), $-(\alpha + \gamma + \mu)$, $-(\tau + \alpha + \delta + \mu)$, plus eigen-values of FV^{-1}.

According to the next-generation framework, the eigenvalues lie strictly in the left half of the complex plane if and only if $R_c< 1$.

Hence, E_0 is locally asymptotically stable when $R_c <1$ and insecurity dies out. Otherwise, if $R_c> 1$, insecurity persists. This establishes a clear quantitative policy threshold for Northern Nigeria.

3.7 Global Dynamics Around the Insecurity-Free Equilibrium

We establish the global asymptotic stability of the insecurity-free equilibrium through an appropriately constructed Lyapunov functional.

Theorem 3:

If R < 1, then the insecurity-free equilibrium E_0 is globally asymptotically stable in the feasible region Ω.

Proof:

Since insecurity is driven by the infective compartmentsI and A, we define the Lyapunov function:

$$L(I,A) = I + \frac{\alpha+\gamma+\mu}{\tau+\alpha+\delta+\mu}A \qquad (16)$$

With $L \geq 0$ for all $I, A \geq 0$, $L = 0$ if and only if $I = A = 0$.

Differentiating equation (16) we have;

$$\frac{dL}{dt} = \frac{dI}{dt} + \frac{\alpha+\gamma+\mu}{\tau+\alpha+\delta+\mu}\frac{dA}{dt}. \qquad (17)$$

Substituting equation(2), (3) into (17), we have;

$$\frac{dL}{dt} = \beta(1+k)(I+A)S - (\alpha+\gamma+\mu)I + \frac{\alpha+\gamma+\mu}{\tau+\alpha+\delta+\mu}\left[\alpha I - (\tau+\alpha+\delta+\mu)A\right]. \qquad (18)$$

Simplifying (18) and grouping the terms (I, A), we have;

Terms involving I;

$$-(\alpha+\gamma+\mu)I + \frac{\alpha+\gamma+\mu}{\tau+\alpha+\delta+\mu} = -(\alpha+\gamma+\mu)\left(1 - \frac{\alpha}{\tau+\alpha+\delta+\mu}\right)I.$$

$$= -\frac{(\alpha+\gamma+\mu)(\tau+\delta+\mu)}{\tau+\alpha+\delta+\mu}I.$$

Terms involving A;

$$-\frac{\alpha+\gamma+\mu}{\tau+\alpha+\delta+\mu}(\tau+\alpha+\delta+\mu)A = -(\alpha+\gamma+\mu)A$$

Bounding the susceptible population from the invariant region, we have;

$$S(t) \leq \frac{\Lambda}{\mu}$$

Hence,

$$\beta(1+k)(I+A)S \leq \beta(1+k)\frac{\Lambda}{\mu}(I+A).$$

$$\frac{dL}{dt} \leq (\alpha+\gamma+\mu)\left[R_c - 1\right](I+A)$$

Where;

$$R_c = \frac{\beta(1+k)\Lambda}{\mu(\alpha+\gamma+\mu)}\left(1+\frac{\alpha}{\tau+\alpha+\delta+\mu}\right)$$ is the control reproduction number.

When $R_c < 1$, we have,

$\frac{dL}{dt} \le 0$, with equality if and only if I = A = 0. The largest invariant set where $\frac{dL}{dt} = 0$ is

$$\{(S, I, A, E, T, R) \in \Omega : I = A = 0\}.$$

By applying LaSalle's Invariance Principle, it is shown that all admissible solution trajectories converge asymptotically to the insecurity-free equilibrium. Consequently, when the control reproduction number is less than one, the insecurity-free equilibrium is globally asymptotically stable. This result demonstrates that sustained insecurity can be eradicated through coordinated institutional reform, strengthened elite accountability, and effective technological regulation.

4 Endemic Equilibrium Analysis

4.1 Endemic (Insecurity-Persistent) Equilibrium

The endemic equilibrium describes a condition where insecurity does not disappear but instead persists over time in the population, that is $I^* > 0$ and $A^* > 0$. Let $E_1 = (S^*, I^*, A^*, E^*, T^*, R^*)$ be the endemic equilibrium, where all derivatives are zero with σ = 0.

$$0 = \Lambda - \beta(1+k)(I+A)S - \mu S, \quad (19)$$

$$0 = \beta(1+k)(I+A)S - (\alpha+\gamma+\mu)I, \quad (20)$$

$$0 = \alpha I + \sigma - (\tau+\alpha+\delta+\mu)A, \quad (21)$$

$$0 = \alpha A - \mu E, \quad (22)$$

$$0 = \tau A - \mu T, \quad (23)$$

$$0 = \gamma I + \delta A - \mu R. \quad (24)$$

From (21), $A^* = \frac{\alpha}{\tau+\alpha+\delta+\mu} I^*$ ....(25)

From (20), $0 = \beta(1+k)(I^* + A^*)S^* - (\alpha+\gamma+\mu)I^*$

$$\beta(1+k)(I^* + A^*)S^* = (\alpha+\gamma+\mu)I^* \quad \text{........(26)}$$

Divide through (25) by I^*, we have;

$$\beta(1+k)(1+\frac{A^*}{I^*})S^* = (\alpha+\gamma+\mu). \quad \text{.......... (27)}$$

Substitute (25) into (27), we have;

$$\beta(1+k)\left(1+\frac{\alpha}{\tau+\alpha+\delta+\mu}\right)S^* = (\alpha+\gamma+\mu).$$

Thus,

$$S^* = \frac{\alpha+\gamma+\mu}{\beta(1+k)\left(1+\frac{\alpha}{\tau+\alpha+\delta+\mu}\right)}. \qquad \text{........} \qquad (28)$$

From equation (19),

$$0 = \Lambda - \beta(1+k)(I+A)S - \mu S$$

Solving for I^* gives;

$$I^* = \frac{\Lambda - \mu S^*}{\alpha+\gamma+\mu} \qquad (29)$$

Put equation (28) into (29) yield,

$$I^* = \frac{\Lambda}{\alpha+\gamma+\mu} - \frac{\mu(\tau+\alpha+\delta+\mu)}{\beta(1+k)(\tau+2\alpha+\delta+\mu)}$$

From (22);

$$E^* = \frac{\alpha}{\mu}A^*$$

From (23);

$$T^* = \frac{\tau}{\mu}A^*$$

From (24);

$$R^* = \frac{\gamma I^* + \delta A^*}{\mu}$$

The existence of the endemic equilibrium is guaranteed when $R_c$exceeds unity. The final expressions of the endemic equilibrium are; $S^* = \frac{\alpha+\gamma+\mu}{\beta(1+k)\left(1+\frac{\alpha}{\tau+\alpha+\delta+\mu}\right)}$, $I^* = \frac{\Lambda}{\alpha+\gamma+\mu} - \frac{\mu(\tau+\alpha+\delta+\mu)}{\beta(1+k)(\tau+2\alpha+\delta+\mu)}$,

$A^* = \frac{\alpha}{\tau+\alpha+\delta+\mu}I^*$, $E^* = \frac{\alpha}{\mu}A^*$, $T^* = \frac{\tau}{\mu}A^*$, $R^* = \frac{\gamma I^* + \delta A^*}{\mu}$.

The endemic equilibrium arises only under the condition that the control reproduction number is above one, confirming that insecurity in Northern Nigeria is structurally sustained by elite influence, technological reinforcement, and institutional bias.

4.2 Global Stability Analysis of the Endemic Equilibrium

Theorem 4:

The endemic equilibrium remains globally asymptotically stable in the positively invariant region whenever R_c is greater than one.

Proof:

Our analysis confirms global asymptotic stability of the endemic equilibrium $E_1 = (S^*, I^*, A^*, E^*, T^*, R^*)$ whenever the control reproduction number R_c.

Since the dynamics of E, T, R are linearly dependent on A and I, global stability is determined by the core subsystem:

$$\frac{dS}{dt} = \Lambda - \beta(1+k)(I + A)S - \mu S \,,$$

$$\frac{dI}{dt} = \beta(1+k)(I + A)S - (\alpha + \gamma + \mu)I \,,$$

$$\frac{dA}{dt} = \alpha I + \sigma - (\tau + \alpha + \delta + \mu)A \,.$$

Let (S^*, I^*, A^*) denote the endemic equilibrium of this subsystem. Constructing a Volterra–type Lyapunov function, we have;

$$L(S, I, A) = \left(S - S^* - S^* In \frac{S}{S^*} \right) + \left(I - I^* - I^* In \frac{I}{I^*} \right) + \left(A - A^* - A^* In \frac{A}{A^*} \right) \quad (30)$$

With $L \geq 0$ for all $S, I, A > 0$ and $L = 0 \Leftrightarrow S, I, A = (S^*, I^*, A^*)$.

Taking the derivative along trajectories of (30), we have,

$$\frac{dL}{dt} = \left(1 - \frac{S^*}{S}\right)\frac{dS}{dt} + \left(1 - \frac{I^*}{I}\right)\frac{dI^*}{dt} + \left(1 - \frac{A^*}{A}\right)\frac{dA^*}{dt} \qquad (31)$$

Substituting equation (1) to (3) into (31), we have;

$$\frac{dL}{dt} = \left(1 - \frac{S^*}{S}\right)\left[\Lambda = \beta(1+k)(I + A)S - \mu S\right] + \left(1 - \frac{I^*}{I}\right)\left[\beta(1+k)(I + A)S - (\alpha + \gamma + \mu)I\right] + \left(1 - \frac{A^*}{A}\right)\left[\alpha I - (\tau + \alpha + \delta + \mu)A\right]$$

After algebraic simplification, we obtain: $\frac{dL}{dt} \leq 0$ and $L = 0 \Leftrightarrow S = S^*, I = I^*, A = A^*$.

Thus, no other invariant set exists where $\frac{dL}{dt} = 0$. Owing to the positive invariance and bounded nature of the admissible region, the Lyapunov function L is radially unbounded at $\frac{dL}{dt} \leq 0$. Using LaSalle's Invariance Principle, it follows that all trajectories emanating from the interior of the admissible region satisfy:

$\left(S(t), I(t), A(t)\right) \rightarrow (S^*, I^*, A^*)$ as $t \rightarrow \infty$.

Therefore, the endemic equilibrium E_1 of the insecurity model is globally asymptotically stable whenever $R_c > 1$.

5 Sensitivity Analysis and Numerical Simulations

5.1 Sensitivity Analysis of the Control Reproduction Number

From equation (15a), the control reproduction is given by

$$R_c = \frac{\beta(1+k)\Lambda}{\mu(\alpha+\gamma+\mu)}\left(1+\frac{\alpha}{\tau+\alpha+\delta+\mu}\right)$$

This threshold quantifies the expected number of new insecurity obtained from the susceptible population when one insecure individual is introduced into a largely insecure-free environment. With respect to the parameter p, the normalized forward sensitivity index of R_c is given by;

$$\gamma_p^{R_c} = \frac{\partial R_c}{\partial p} \cdot \frac{p}{R_c}$$

Positive values mean that increasing p increases R_c (worsens insecurity), and negative values mean the opposite.

Table 4 shows the sensitivity result of each parameter values. The strongest driver is β which shows that for every 1% increase in baseline insecurity transmission (frequency of violent interaction) increases $R_c$by **1%**. The structural bias (k) in institutions increases insecurity persistence with a value of 0.38. Therefore, constitutional weaknesses are significant. Elite influence (α) accounts for about 20 % sensitivity elites shaping insecurity dynamics (politically or economically) matter. Recovery (γ) has a strong negative effect, thus increasing reintegration programs significantly reduces persistence. Technology (τ) and reform (δ) parameters have modest negative effects, indicating targeted reforms and technology disruption can help. The contributions of the model parameters to the control reproduction number are presented in Figure 2 to figure 7.

5.2 Numerical Simulation of the Global Stability Analysis of the Endemic Equilibrium Point

Numerical simulations are carried out in this section to verify the analytical results of the proposed insecurity model, examine the qualitative dynamics of the system, assess equilibrium stability, and evaluate the impact of key elite, technology, and constitutional bias parameters on insecurity dynamics in Northern Nigeria. This system admits an endemic equilibrium $E_1 = (I^*, A^*,)$ when $R_c > 1$. Table 3 presents the baseline parameter values for the simulations, which result in $R_c \approx 1.84 > 1$, signifying the existence of a unique endemic equilibrium. The focus was placed on the key insecurity-driving compartments (I(t) and A(t)). To test global stability, simulations were initiated from arbitrary interior initial conditions as shown in figure 8 and figure 9 and explain in detail in chapter 5 with total population normalized to unity. The trajectories are represented by blue, red, and green solid plots that converge towards the equilibrium point as the time approaches infinity.

Table 3 Parameter value of R_c

Parameter	Value	Source
β	0.60	Assumed
k	0.50	[5].
α	0.30	[8].
γ	0.40	Assumed.
τ	0.20	[2].
δ	0.10	Assumed.
μ	0.02	Assumed.
Λ	10000	Assumed.

Table 4 Sensitivity Analysis Result

Parameter	Sensitivity index
β	+1.0
k	+0.38
α	+0.20
γ	-0.29
τ	-0.12
δ	-0.08

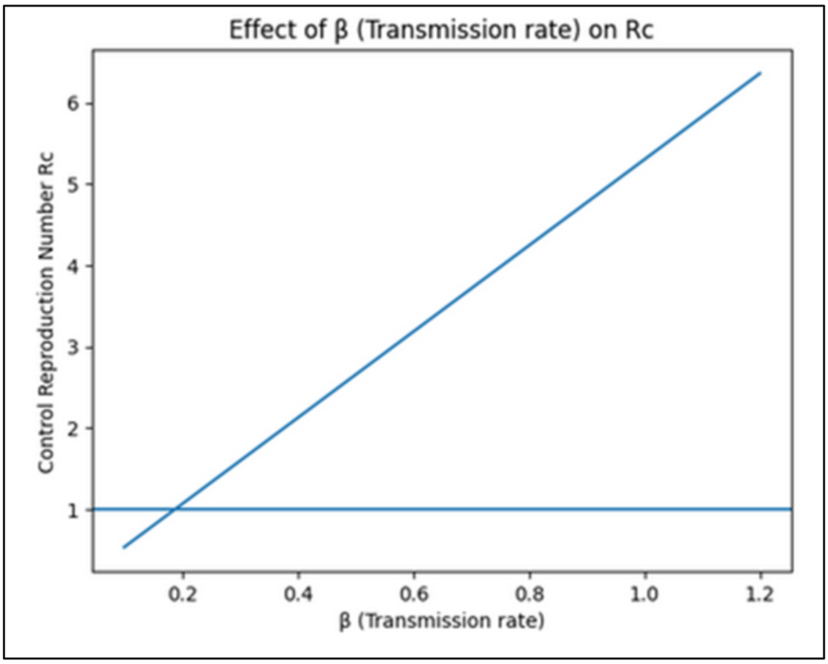

Figure 2 Impact of the Parameter β on R_c.

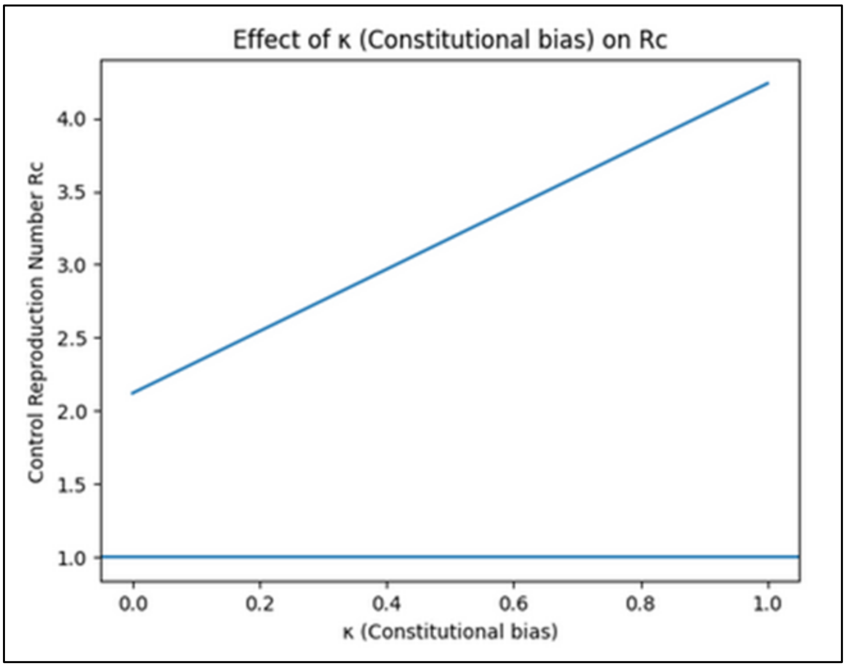

Figure 3 Impact of the Parameter k on R_c.

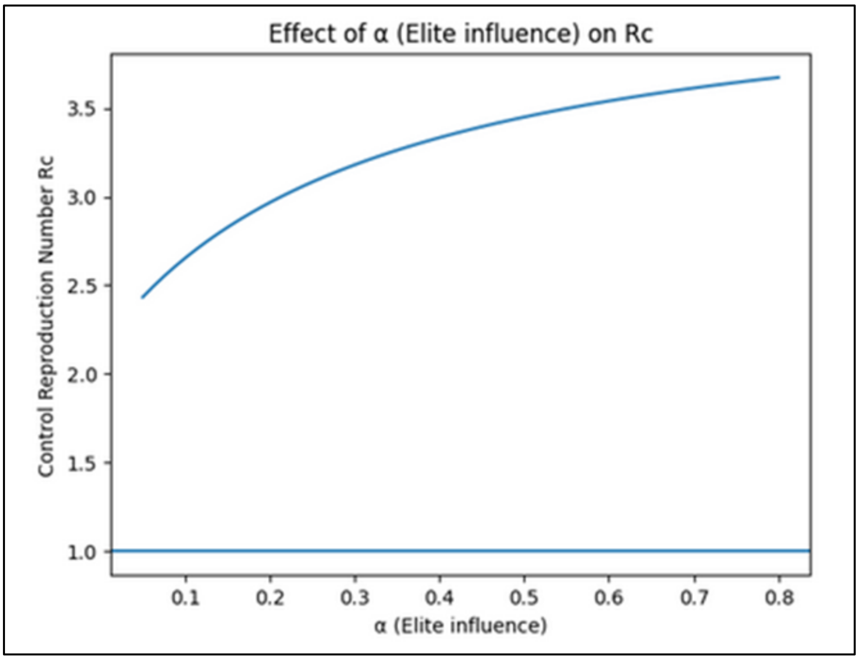

Figure 4 Impact of the Parameter α on R_c.

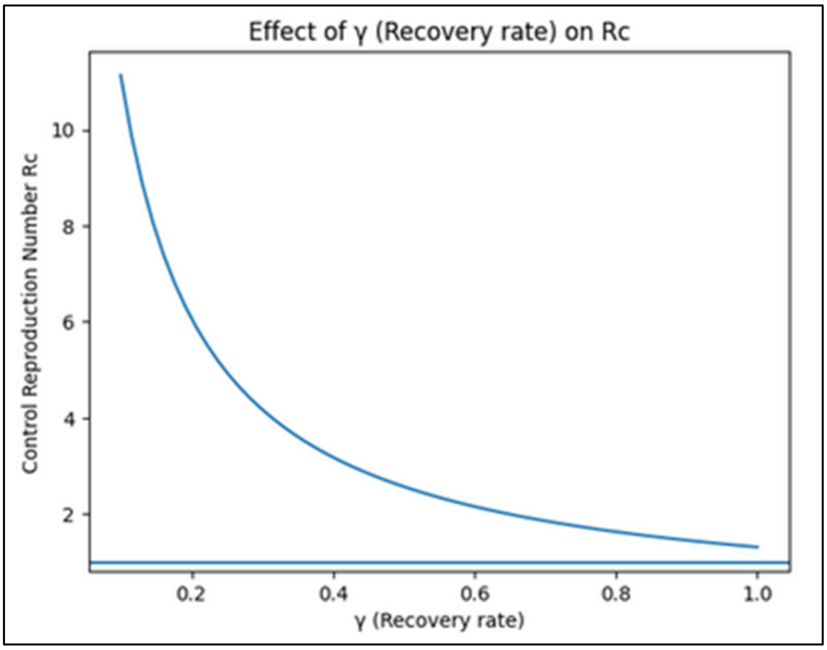

Figure 5 Impact of the Parameter γ on R_c.

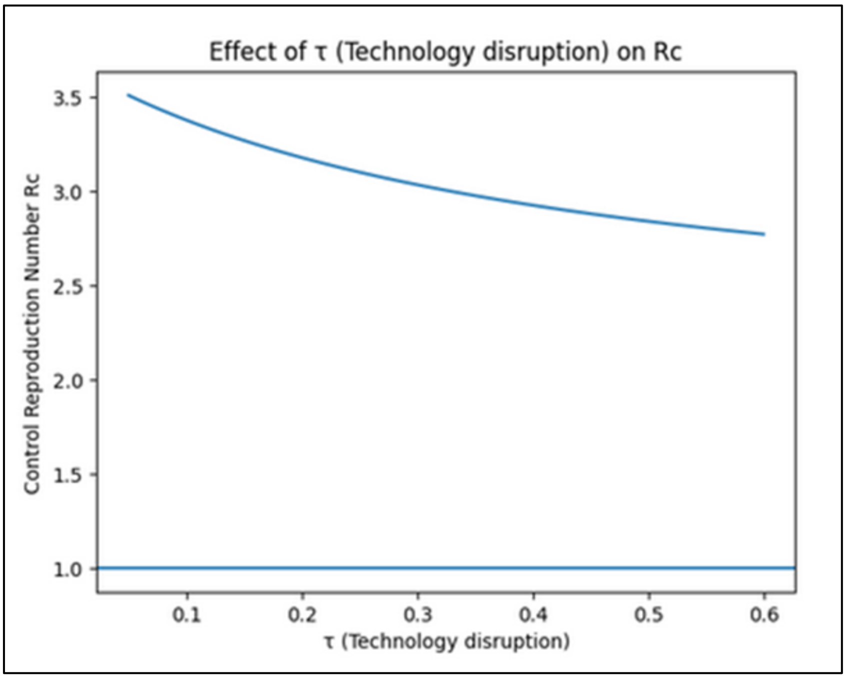

Figure 6 Impact of the Parameter τ on R_c.

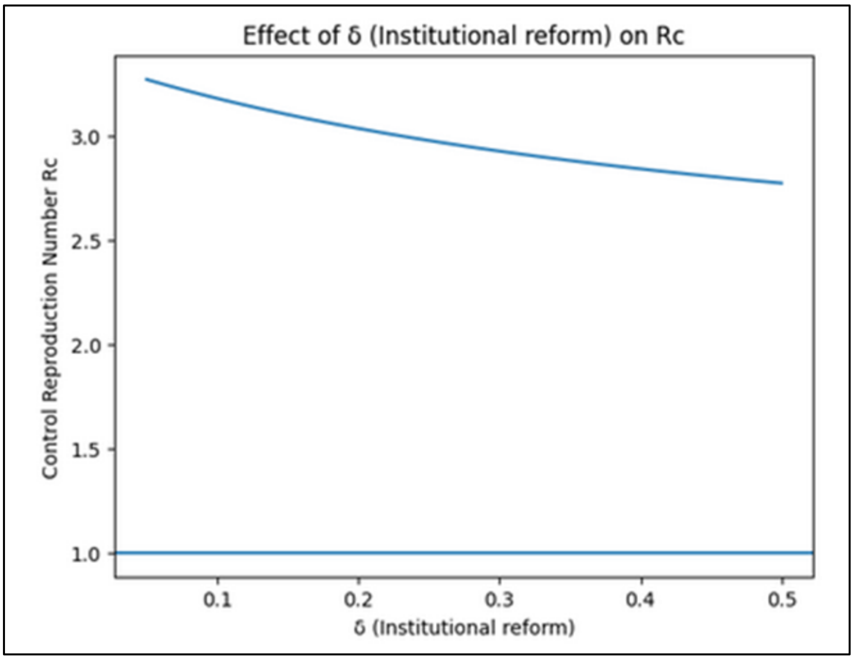

Figure 7 Impact of the Parameterf δ on R_c.

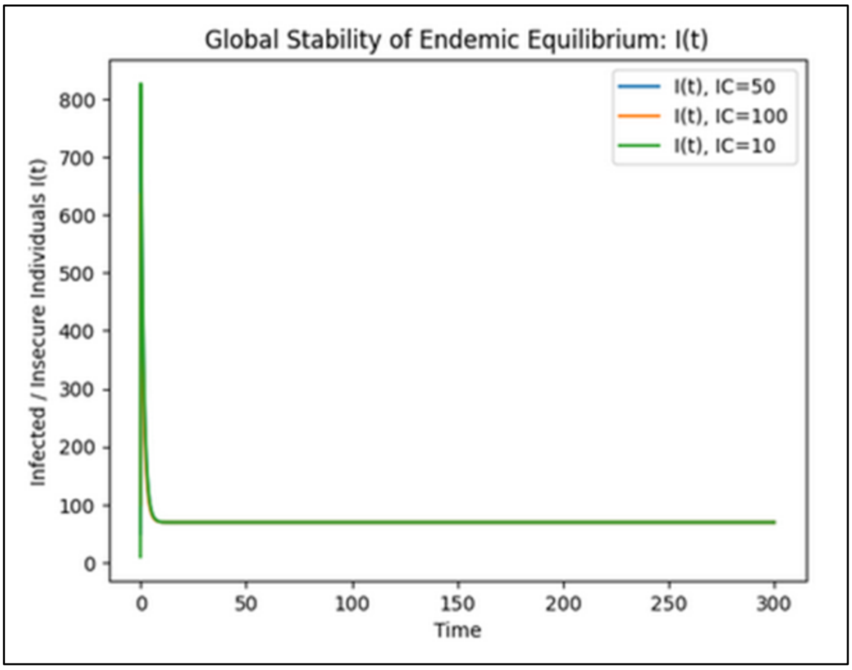

Figure 8 Illustration of the Global Stability of the Equilibrium I(t)

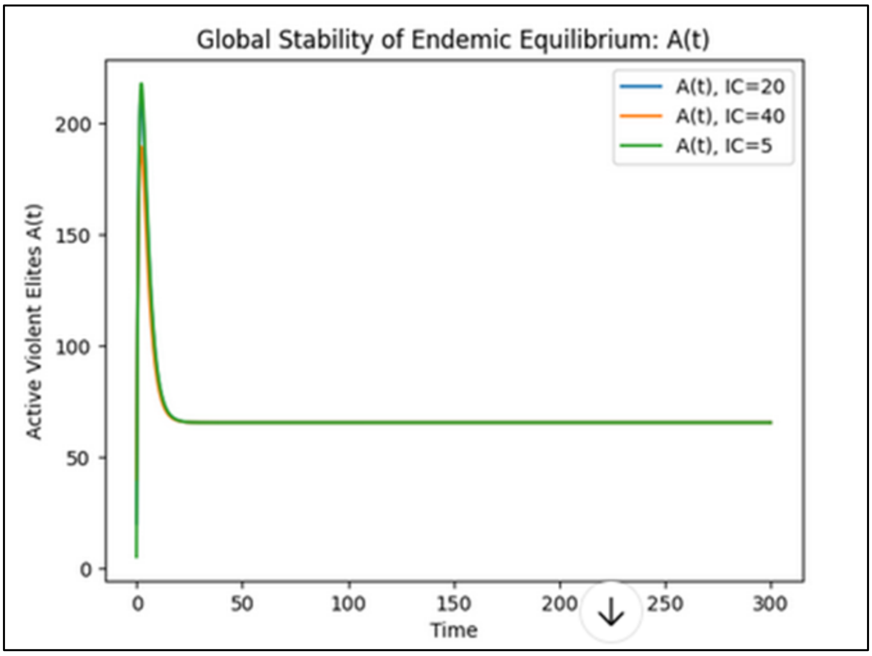

Figure 9 Illustration of the Global Stability of the Equilibrium A(t)

6 Discussion

6.1 Effect of Transmission Rate β on R_c

6.1.1 Observation from the graph (Figure 2)

R_c increases linearly and monotonically with β. A small increase in β rapidly pushes R_c above the critical threshold $R_c = 1$.

6.1.2 Interpretation

The parameter β captures the intensity of interaction between susceptible individuals and active violent actors (bandits, insurgents, militias). In Northern Nigeria, high population mobility, porous borders, and communal interactions increase exposure to violent recruitment and spillover effects.

6.1.3 Policy implication

Reducing contact intensity through improved surveillance, border control, and early-warning intelligence is the most effective single intervention, since β has the strongest influence on insecurity persistence.

6.2 Effect of Constitutional Bias κ on Rc

6.2.1 Observation from the graph (Figure 3)

R_c increases steadily with κ. Even moderate levels of constitutional bias keep $R_c > 1$.

6.2.2 Interpretation

The parameter κ represents perceived political exclusion, unequal resource allocation, and governance imbalance. In Northern Nigeria, long-standing grievances related to representation and policy bias magnify insecurity by legitimizing violent resistance narratives.

6.2.3 Policy implication:

Governance reforms and inclusive constitutional frameworks can significantly weaken insecurity, even without military escalation.

6.3 Effect of Elite Influence α on Rc

6.3.1 Observation from the graph (Figure 4)

R_c increases with α, but the curve saturates at higher values. Initial increases in α cause sharp rises in insecurity.

6.3.2 Interpretation

Elite sponsorship, protection, or financing of violent groups strongly fuels insecurity at early stages. However, saturation reflects structural limits once recruitment channels are fully active, further elite influence yields diminishing marginal effects.

6.3.3 Policy implication

Targeting elite enablers through accountability, financial tracking, and political sanctions is crucial in the early containment of insecurity.

6.4 Effect of Recovery Rate γ on Rc

6.4.1 Observation from the graph (Figure 5)

R_c decreases sharply as γ increases. The curve is strongly nonlinear, with steep decline at low γ.

6.4.2 Interpretation

The recovery rate represents de-radicalization, reintegration, amnesty programs, and disengagement from violence. In Northern Nigeria, successful rehabilitation programs dramatically reduce the pool of active violent actors.

6.4.3 Policy implication

Investment in reintegration programs yields high returns, particularly when recovery capacity is initially weak.

6.5 Effect of Technology Disruption τ on R_c

6.5.1 Observation from the graph (Figure 6)

R_c decreases monotonically with increasing τ. Decline is gradual but consistent.

6.5.2 Interpretation

The parameter τ measures disruption of communication, financing, and coordination technologies used by violent groups (e.g., encrypted messaging, digital payments). In Northern Nigeria, technology facilitates coordination across vast terrains.

6.5.3 Policy implication

Cyber-monitoring, fintech regulation, and communication surveillance are essential complementary tools to physical security measures.

6.6 Effect of Institutional Reform δ on Rc

6.6.1 Observation from the graph (Figure 7)

R_c decreases slowly as δ increases. Impact is weaker compared to γ or τ.

6.6.2 Interpretation

Institutional reform improves justice delivery, security response time, and rule of law. However, reforms act indirectly and over longer horizons, explaining the gradual effect.

6.6.3 Policy implication

Institutional reform is necessary for long-term stability but must be combined with short-term interventions to cross the insecurity threshold.

6.7 Dynamics of Insecure Individuals I(t)

6.7.1 Observation from the graph (Figure 8)

For widely different initial conditions, all solution trajectories initially diverge, then rapidly converge, finally settle at the same positive equilibrium value.

6.7.2 Interpretation

This confirms that $\lim_{t\to\infty} I(t) = I^*$ for all admissible initial conditions. Hence, within the 'I' component, the endemic equilibrium maintains global asymptotic stability.

6.7.3 Insecurity meaning

Regardless of how small or large insecurity initially is, once $R_c > 1$, the system settles into a persistent insecurity state. Short-term military or security successes do not eliminate insecurity unless parameters are changed structurally.

6.8 Dynamics of Active Violent Elites A(t)

6.8.1 Observation from the graph (Figure 9)

Initial spikes represent early elite mobilization, political or economic shocks. All trajectories converge to a unique positive equilibrium level.

6.8.2 Interpretation:

$\lim_{t\to\infty} A(t) = A^*$ independent of initial elite strength.

6.8.3 Insecurity meaning

Elite-driven violence stabilizes at a **non-zero level**, sustained by recruitment, technology, weak institutions. This validates the structural nature of insecurity in Northern Nigeria.

Therefore, the numerical simulations confirm that if $R_c>1$, the endemic equilibrium E_1 is globally asymptotically stable. That is, $\lim_{t\to\infty}\left(S, I, A, E, T, R\right) = \left(S^*, I^*, A^*, E^*, T^*, R^*\right)$ for all admissible initial conditions.

6.9 Policy Interpretation for Northern Nigeria

Security-only interventions cannot eradicate insecurity once it becomes endemic. Persistence is driven by elite incentives, constitutional bias, and technological coordination. Only structural reforms that reduce β, κ, and α, or significantly increase γ, τ, and δ, can shift the system back to the insecurity-free equilibrium.

7 Conclusion

This work presents a novel mathematical framework for understanding insecurity as a self-reinforcing social contagion influenced by elite behaviour, technology, and constitutional structures.Results from the analytical study show that the insecurity-free equilibrium exists and maintains local asymptotic stability for $R_c<1$. Conversely, when $R_c>1$, the system has a single endemic equilibrium that exhibits global asymptotic stability, indicating persistent insecurity regardless of initial conditions. Sensitivity analysis identifies transmission intensity and constitutional bias as dominant drivers, while recovery and institutional reform offer the strongest mitigation pathways. Numerical simulations corroborate the theoretical findings and highlight the long-term consequences of structural governance failures. Overall, the study demonstrates that insecurity in Northern Nigeria is not merely episodic but systemic, requiring coordinated policy responses beyond short-term security operations.

7.1 Recommendations

- Policies aimed at reducing structural bias and political exclusion should be prioritized, as constitutional imbalance significantly amplifies insecurity persistence.
- Strengthening accountability mechanisms to disrupt elite sponsorship and protection of violent actors is critical for reducing recruitment and operational capacity.

- Expanding de-radicalization, rehabilitation, and community reintegration programs yields substantial reductions in long-term insecurity.
- Monitoring digital communication and financial technologies used by violent groups can weaken coordination and reduce operational efficiency.
- Military responses should be complemented with socio-economic and institutional interventions to effectively push R_c below unity.

7.1.1 Further Work

- Future research may extend the model by incorporating;
- Stochastic effects to capture uncertainty and random shocks such as sudden attacks or political crises.
- Spatial dynamics to study cross-border spillovers and regional heterogeneity within Northern Nigeria.
- Time-dependent controls to evaluate optimal intervention strategies using optimal control theory.
- Data-driven calibra tion using conflict event databases to enhance predictive accuracy.
- Game-theoretic frameworks to explicitly model strategic interactions between elites, the state, and communities.

Compliance with ethical standards

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Disclosure of conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Statement of Ethical Approval

The study utilized secondary data obtained from publicly available sources to inform the mathematical model of insecurity dynamics in Northern Nigeria. No human or animal subjects were directly involved and ethical approval was not required.

Statement of Informed Consent

Informed consent was not required as the study was based exclusively on publicly available and anonymized secondary data.

References

[1] Adebayo, A. A. (2014). Implications of Boko Haram terrorism on national development in Nigeria. Journal of Social Sciences, 40(3), 213–223.

[2] Conway, M. (2017). Determining the role of the internet in violent extremism and terrorism. Studies in Conflict & Terrorism, 40(1), 77–98. https://doi.org/10.1080/1057610X.2016.1157408

[3] Epstein, J. M. (2002). Modeling civil violence: An agent-based computational approach. Proceedings of the National Academy of Sciences of the United States of America, 99(Suppl. 3), 7243–7250. https://doi.org/10.1073/pnas.092080199

[4] Kaldor, M. (2013). New and old wars: Organized violence in a global era (3rd ed.). Polity Press.

[5] Matfess, H. (2017). Institutionalizing instability: The constitutional roots of insecurity in Nigeria's Fourth Republic. International Journal of Security & Development, 6(1), Article 12. https://doi.org/10.5334/sta.487

[6] Mosca, G. (1939). The ruling class. McGraw-Hill.

[7] Mustapha, A. R. (2014). Ethnic structure, inequality and governance of the public sector in Nigeria (UNRISD Working Paper No. 2014-24). United Nations Research Institute for Social Development.

[8] Ogundiya, I. S., & Amzat, J. (2019). The political economy of insecurity in Nigeria. International Journal of Criminology and Sociology, 8, 473–485. https://doi.org/10.6000/1929-4409.2019.08.41

[9] Pareto, V. (1935). The mind and society. Harcourt, Brace & Company.

[10] Suberu, R. T. (2009). Federalism in Africa: The Nigerian experience in comparative perspective. Ethnopolitics, 8(1), 67–86. https://doi.org/10.1080/17449050902749080

[11] Turchin, P. (2003). Historical dynamics: Why states rise and fall. Princeton University Press.

[12] Van den Driessche, P., & Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Mathematical Biosciences, 180(1-2), 29-48. https://doi.org/10.1016/S0025-5564(02)00108-6