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Modelling Youth Gambling Behaviour in Nigeria: The Roles of Digital Exposure, Debt Accumulation, Relapse, and Policy Control

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Abstract

Youth gambling has expanded rapidly in Nigeria alongside increased access to digital betting platforms, yet its long-term social and behavioural dynamics remain poorly quantified. This study develops a deterministic compartmental model to examine the progression of youth gambling by incorporating digital exposure, escalation of betting behavior, financial debt accumulation, recovery with relapse, and regulatory enforcement. The model is shown to be mathematically well-defined, with positive and bounded solutions. A threshold quantity is derived to characterize gambling persistence, and its role in governing system dynamics is rigorously analyzed. Sensitivity analysis identifies digital exposure and regulatory enforcement as the most influential drivers of gambling prevalence. Numerical simulations with baseline parameters reveal the existence of a stable endemic gambling state under current conditions, marked by sustained levels of active and problem gambling and growing debt burden. The findings highlight the necessity of structural interventions targeting exposure control and enforcement to prevent long-term entrenchment of youth gambling in Nigeria.

Keywords: Compartmental Modeling; Digital Betting Platforms; Financial Debt Accumulation; Gambling Persistence Threshold; Regulatory Enforcement; Youth Gambling

1. Introduction

In recent years, online sports betting has become a dominant and fast-growing segment of gambling activity in Nigeria, largely driven by increased internet penetration, mobile technology, and aggressive digital advertising targeting youths. While betting companies are often promoted as sources of entertainment and employment, mounting evidence indicates that excessive gambling among youths is associated with financial distress, academic underperformance, mental health challenges, and social instability [2]; [7]. In Nigeria, weak regulatory enforcement and easy access to online betting platforms have further intensified youth participation in gambling activities [12].

Mathematical modelling has increasingly been used to study the evolution of social behaviours, including gambling, by treating participation and addiction as dynamic processes within a population. Existing gambling models have largely focused on initiation and addiction dynamics, with limited consideration of digital exposure, relapse mechanisms, and financial debt, which are critical drivers of gambling persistence in developing economies [15]; [1]. In particular, the role of debt in reinforcing gambling behavior and undermining recovery efforts remains underexplored in the Nigerian context.

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Most existing mathematical models of gambling behaviour (including [6]; [9]; Cognitive Behavioural Therapy-based models) share common features such as peer-influenced initiation, progression to problem gambling, and recovery and relapse dynamics. However, critical Nigeria-specific and modern drivers of gambling are not explicitly captured, such as digital/algorithmic exposure (mobile apps, social media ads, bonus notifications), debt accumulation and financial distress (which independently worsen addiction), regulatory and policy enforcement effects (advert bans, betting limits, age verification), economic shocks (unemployment, inflation spikes), and household spillover effects (family financial stress feeding back into relapse).

To address these gaps, we develop an extended compartmental model for youth gambling in Nigeria, integrating digital exposure, debt accumulation, relapse, and policy enforcement in a unified framework and derive analytical thresholds and stability conditions to inform evidence-based policy decisions.

2. Model Formulations and Assumptions

2.1. The Gambling Model Formulation

The model divides the youth population into six interacting compartments according to gambling status, financial debt, and recovery. Transitions between compartments capture key behavioural and socio-economic mechanisms driving youth gambling in Nigeria.

Susceptible youths, $S(t)$, are individuals who have not engaged in gambling. They enter the population at rate Λ, become digitally exposed through online advertising and peer influence at rate λS, and exit naturally at rate μS. Recovered youths who lose immunity return to susceptibility at rate ϕR.

Digitally exposed youths, $E(t)$, have encountered gambling content but have not yet placed bets. They arise from the susceptible class at rate λS and through relapse of recovered individuals at rate σR. Exposed youths progress to active gambling at rate αE and leave the class through natural exit at rate μE.

Active gamblers, $B(t)$, are youths who place bets but are not yet compulsive gamblers. They enter from the exposed class at rate αE, progress to problem gambling at rate δB, and recover through awareness or self-control at rate γB. Natural exit occurs at rate μB.

Problem gamblers, $P(t)$, exhibit addictive gambling behaviour. They enter from active gambling at rate δB, accumulate financial debt at rate ρP, and recover through treatment or intervention at rate κP. Natural exit occurs at rate μP.

Indebted gamblers, $D(t)$, represent individuals whose gambling has resulted in substantial financial debt. They arise from the problem gambling class at rate ρP, recover through counseling, debt relief, or policy-supported interventions at rate ηD, and exit naturally at rate μD.

Recovered youths, $R(t)$, consist of individuals who have temporarily stopped gambling. Recovery occurs from active gambling, problem gambling, and indebtedness at rates γB, κP, and ηD, respectively. Recovered youths may relapse into digital exposure at rate σR or return to susceptibility due to loss of immunity at rate ϕR. Natural exit occurs at rate μR.

2.2. The Gambling Model Assumptions

- The youth population is partitioned into mutually exclusive compartments according to gambling status, financial debt, and recovery.
- Youths interact homogeneously, and gambling behaviour spreads through digital exposure and peer influence.
- Digital advertising and online betting platforms are the primary drivers of gambling initiation.
- Financial debt reinforces gambling persistence and accelerates progression to more severe gambling states.
- Recovery from gambling is temporary, and relapse may occur due to continued digital exposure.
- Policy enforcement reduces gambling initiation and progression but is imperfect.
- Individuals exit each compartment through natural processes at a constant rate.
- All model parameters are assumed constant over the study period.

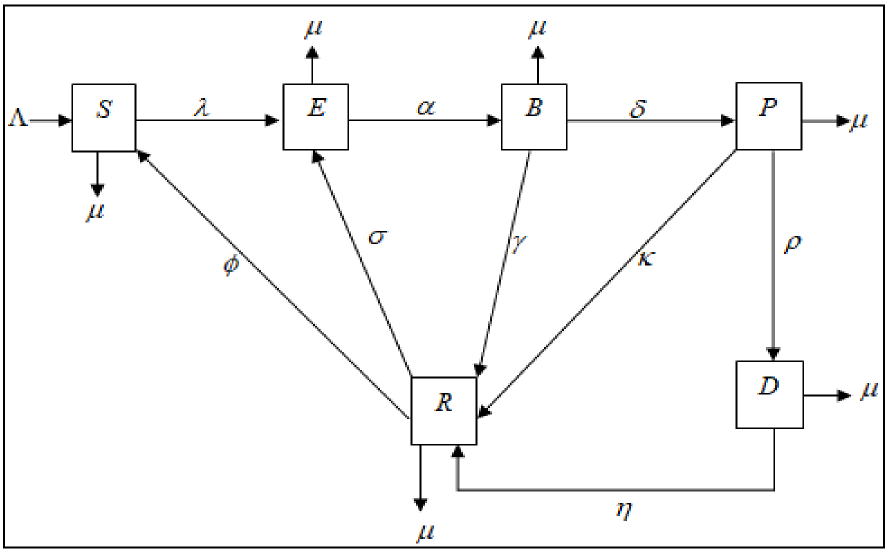

Figure 1 Structural diagram illustrating transitions within the gambling model

3. Mathematical Analysis of the Model

3.1. Model equations for gambling behavior dynamics

Based on the structural diagram of the gambling framework, the following system of equations is derived.

$$\frac{dS}{dt} = \Lambda - \lambda S + \phi R - \mu S \,, \tag{1}$$

$$\frac{dE}{dt} = \lambda S + \sigma R - (\alpha + \mu)E \,, \tag{2}$$

$$\frac{dB}{dt} = \alpha E - (\delta + \gamma + \mu)B \,, \tag{3}$$

$$\frac{dP}{dt} = \delta B - (\kappa + \rho + \mu)P \,, \tag{4}$$

$$\frac{dD}{dt} = \rho P + (\eta + \mu)D \,, \tag{5}$$

$$\frac{dR}{dt} = \gamma B + \kappa P + \eta D - (\phi + \sigma + \mu)R \,. \tag{6}$$

We define the force of gambling exposure as

$$\lambda(t) = (1 - C)\left(\omega + \beta \frac{B(t) + P(t)}{N(t)}\right),$$

where C ∈ [0,1] represents the level of policy enforcement (advertisement restriction, age verification, self-exclusion).

Table 1 State variables of the gambling model

State variables	Interpretation
$S(t)$	Susceptible (non-gambling) youths.
$E(t)$	Digitally exposed youths (advertisements, peer influence)
$B(t)$	Active gamblers
$P(t)$	Problem/compulsive gamblers
$D(t)$	Indebted gamblers
$R(t)$	Recovered youths

Table 2 Model parameters of the gambling model

Model parameters	Interpretations
Λ	Recruitment rate into the youth population.
μ	Natural exit rate (aging out, migration).
ω	Digital advertisements exposure rate.
β	Peer-induced gambling influence.
C	Policy enforcement strength ($0 \le C \le 1.$)
α	Progression rate from exposure to gambling.
δ	Transition rate from gambling to problem gambling.
γ	Recovery rate of gamblers.
ρ	Debt accumulation rate.
k	Recovery rate of problem gamblers.
η	Recovery rate of indebted gamblers.
ϕ	Loss of immunity (return to susceptibility).
σ	Relapse rate due to digital exposure.

3.2. Positivity of the Gambling System

Theorem 1:

The gambling model (1) to (6) is positive if solutions $\{S(t), E(t), B(t), P(t), D(t), R(t),\} \in R_+^6$ generated from non-negative initial states $\{S(0), E(0), B(0), P(0), D(0), R(0)\} \in R_+^6$ remain non-negative throughout the time interval t > 0.

Proof: The proof of compartment-wise positivity relies on the fact that solutions originating in the non-negative region cannot cross the coordinate hyperplanes.

From equation (1); $\frac{dS}{dt} = \Lambda - \lambda S + \phi R - \mu S$. At S = 0, $\frac{dS}{dt} = \Lambda + \phi R \ge 0$

Hence, S(t) cannot cross into negative values.

From equation (2),

At E = 0, $\frac{dE}{dt} = \lambda S + \sigma R \geq 0$. Thus, $E(t) \geq 0$ for all $t \geq 0$.

From equation (3),

At B = 0, $\frac{dB}{dt} = \alpha E \geq 0$. Therefore, $E(t)$ remains non-negative.

From equation (4),

At P = 0, $\frac{dP}{dt} = \delta B \geq 0$. Thus, $P(t) \geq 0$.

From equation (5),

At D = 0, $\frac{dD}{dt} = \rho P \geq 0$. Hence, $D(t) \geq 0$.

From equation (6),

At R = 0, $\frac{dR}{dt} = \gamma B + \kappa P + \eta D \geq 0$. Thus, $R(t) \geq 0$.

Applying the comparison principle, each equation satisfies; inflow terms ≥ 0 and outflow terms proportional to the variable itself. Hence, the vector field *points*inward on the boundary of the non-negative orthant $\Re_+^6$ and solutions remain non-negative for all t ≥ 0.

Hence, all state variables of the extended gambling model remain non-negative for all time, provided the initial conditions are non-negative. Therefore, the model is positively invariant and mathematically consistent and behaviourally admissible $S(t), E(t), B(t), P(t), D(t), R(t) \geq 0$ for all t ≥ 0.

This positivity proof follows standard approaches used in compartmental modelling [17], [5].

3.3. The Invariant Region of the Gambling Model

Theorem 2:

Let $\Omega = \left\{ (S, E, B, P, D, R) \in \Re_+^6 : N(t) \leq \frac{\Lambda}{\mu} \right\}$, where $N = S + E + B + P + D + R.$

Then, the region Ω is positively invariant under the flow of the extended gambling model.

To ascertain the boundedness of the population, we examine the growth of the total population N(t). Summing all the equations (1) to (6) results to

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dE}{dt} + \frac{dB}{dt} + \frac{dP}{dt} + \frac{dD}{dt} + \frac{dR}{dt}. \qquad (7)$$

Substituting the model (1) – (6) equations and simplifying gives

$$\frac{dN}{dt} = \Lambda - \mu(S + E + B + P + D + R), \tag{8}$$

Hence,

$$\frac{dN}{dt} = \Lambda - \mu N \tag{9}$$

Equation (9) is linear and has the explicit solution

$$N(t) = \frac{\Lambda}{\mu} + \left(N(0) - \frac{\Lambda}{\mu}\right)e^{-\mu t} \tag{10}$$

From the solution (10) above, it follows that

$$\lim_{t\to\infty} N(t) \le \frac{\Lambda}{\mu}. \tag{11}$$

If $N(0) \le \frac{\Lambda}{\mu}$ then $N(t) \le \frac{\Lambda}{\mu}$ for all $t \ge 0$.

Thus, the total youth population remains bounded.

From the positivity result, we already have: $S(t), E(t), B(t), P(t), D(t), R(t) \ge 0$ for all t ≥ 0.

Combining this with the population bound: $N(t) \le \frac{\Lambda}{\mu}$, we conclude that solutions starting in Ωnever leave the region Ω.

Therefore, all trajectories of the extended gambling model that start in the biologically feasible region $\Omega = \left\{(S, E, B, P, D, R) \in N(t) \le \frac{\Lambda}{\mu}\right\}$ remain in Ω for all t ≥ 0.

The proof follows standard arguments used in compartmental dynamical systems [17], [8].

3.4. Gambling-Free Steady State of the Model

The Gambling-free steady state of the model refers to the situation in which no individual is involved in gambling or its consequences. Hence, all gambling-related compartments are set to zero. That is,

$$\lambda = (1 - C)\left(\omega + \beta\frac{B + P}{N}\right), \ E_0 = B_0 = P_0 = 0 \tag{12}$$

Recall, $N = S + E + B + P + D + R.$

Substituting (12) into the force of exposure gives

$$\lambda = (1 - C)\omega, \tag{13}$$

since $B_0 = P_0 = 0$.

From equation (1), $$0=\Lambda-\lambda_0 S_0-\mu S_0. \quad (14)$$

Substituting (13) into (14), we have, $0=\Lambda-[(1-C)\omega+\mu]S_0$,

$$S_0=\frac{\Lambda}{\mu+(1-C)\omega}. \quad (15)$$

$$\frac{dD}{dt}=\rho P+(\eta+\mu)D=0\Rightarrow D_0=0.$$

$$\frac{dR}{dt}=-(\phi+\sigma+\mu)R=0\Rightarrow R_0=0.$$

Thus, all equations are satisfied simultaneously.

The gambling-free equilibrium is therefore $E_0=\left(\frac{\Lambda}{\mu+(1-C)\omega},0,0,0,0\right)$. (16)

This means (16) S_0 decreases as digital exposure ω increases. Stronger policy enforcement Cincreases the gambling-free population when C→1 (full enforcement), $S_0\rightarrow\frac{\Lambda}{\mu}$, the maximal youth population.

3.5. Effective Reproduction Number of the Gambling System

The effective reproduction number R_e measures the expected number of new gambling entrants attributable to a single gambler in the presence of regulatory controls and policy measures. The formulation is obtained using next-generation matrix methodology [17]. Gambling transmission occurs through the following gambling compartments:

$$X=(E,B,P)^T,$$

Decomposition into new-gambling and transition terms, we have,

$\frac{dX}{dt}=F(X)-V(X)$, F describes new gambler formation, and V represents transition and exit rates. Therefore, the new gambling terms $F=\begin{pmatrix}(1-C)\beta\frac{S(B+P)}{N}\\0\\0\end{pmatrix}$,

The transition terms $$V=\begin{pmatrix}(\alpha+\mu)E\\(\delta+\gamma+\mu)B-\alpha E\\(\kappa+\rho+\mu)P-\delta B\end{pmatrix}$$

Jacobian matrices at the gambling-free equilibrium is given as, Jacobian of F will be $F = \begin{pmatrix} 0 & (1-C)\beta & (1-C)\beta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, (17)

Where, $\frac{S_0}{N_0} = 1$.

$$V = \begin{pmatrix} \alpha+\mu & 0 & 0 \\ -\alpha & \delta+\gamma+\mu & 0 \\ 0 & -\delta & \kappa+\rho+\mu \end{pmatrix}.$$

By computing the inverse of the matrix V, we obtain;

$$V^{-1} = \begin{pmatrix} \frac{1}{\alpha+\mu} & 0 & 0 \\ \frac{\alpha}{(\alpha+\mu)(\delta+\gamma+\mu)} & \frac{1}{\delta+\gamma+\mu} & 0 \\ \frac{\alpha\delta}{(\alpha+\mu)(\delta+\gamma+\mu)(\kappa+\rho+\mu)} & \frac{\delta}{(\delta+\gamma+\mu)(\kappa+\rho+\mu)} & \frac{1}{\kappa+\rho+\mu} \end{pmatrix} \quad (18)$$

The resulting next-generation matrix, defined as K = FV^{-1} is carried out by multiplying (17) and (18), thus;

$$K = \begin{pmatrix} (1-C)\beta\left[\frac{\alpha}{(\alpha+\mu)(\delta+\gamma+\mu)(\kappa+\rho+\mu)}\right] & k_{12} & k_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Where $$k_{12} = (1-C)\beta\left[\frac{1}{\delta+\gamma+\mu} + \frac{\delta}{(\delta+\gamma+\mu)(\kappa+\rho+\mu)}\right],$$

$$k_{13} = (1-C)\beta\left[\frac{1}{\kappa+\rho+\mu}\right].$$

Because K is upper triangular, its eigenvalues are: $\lambda_1 = (1-C)\beta\frac{\alpha}{(\alpha+\mu)(\delta+\gamma+\mu)}\left(1+\frac{\delta}{\kappa+\rho+\mu}\right)$, $\lambda_2 = 0$, $\lambda_3 = 0$.

Thus, $R_e = \rho(K) = \lambda_1$.

$$R_e = (1-C)\beta\frac{\alpha}{(\alpha+\mu)(\delta+\gamma+\mu)}\left(1+\frac{\delta}{\kappa+\rho+\mu}\right). \quad (19)$$

The effective reproduction number captures the combined effects of digital exposure, peer influence, addiction escalation, financial debt, and policy enforcement on youth gambling dynamics.

3.6. Local stability of the Gambling-Free Steady State

Theorem 3:

The non-gambling equilibrium remains locally attracting for R_e below unity and becomes unstable once R_e exceeds one.

Proof:

The threshold quantity R_e (19) plays a central role in determining the qualitative behaviour of the gambling-free equilibrium. When this threshold remains below unity, small perturbations involving gambling-related states decay over time, causing trajectories to return toward the non-gambling equilibrium. Conversely, once R_e exceeds one, gambling-related compartments gain sufficient momentum to grow from arbitrarily small levels, rendering the gambling-free equilibrium unstable. This threshold behavior reflects the balance between gambling initiation mechanisms driven by exposure and peer influence and removal mechanisms such as recovery and policy enforcement, and it provides a clear criterion for assessing the effectiveness of intervention strategies.

3.7. Global Stability of the Gambling-Free Steady State

We will use a Lyapunov-based approach following [4]. We now define the Lyapunov function as

$$L(E,B,P) = a_1 E + a_2 B + a_3 P \qquad (20)$$

where $a_1, a_2, a_3 > 0$ are constants to be selected.

pick, $a_1 = \dfrac{1}{\alpha+\mu}$, $a_2 = \dfrac{\alpha}{(\alpha+\mu)(\delta+\gamma+\mu)}$, $a_3 = \dfrac{\alpha\delta}{(\alpha+\mu)(\delta+\gamma+\mu)(\kappa+\rho+\mu)}$.

Differentiating equation (24a), we have;

$$\frac{dL}{dt} = a_1\frac{dE}{dt} + a_2\frac{dB}{dt} + a_3\frac{dP}{dt}.$$

$$\frac{dL}{dt} \le \left[(1-C)\beta\frac{\alpha}{(\alpha+\mu)(\delta+\gamma+\mu)}\left(1+\frac{\delta}{\kappa+\rho+\mu}\right)\right](B+P).$$

Therefore,

$$\frac{dL}{dt} \le (R_e - 1)(B+P). \qquad (21)$$

The signs of (21) will be, if $R_e < 1$, then $\dfrac{dL}{dt} \le 0$, and equality holds if and only if $E = B = P = 0$.

The largest invariant set where $\dfrac{dL}{dt} = 0$ is the gambling-free equilibrium E_0.

Therefore, by LaSalle's invariance principle, the gambling free equilibrium is globally asymptotically stable in the feasible region if $R_e < 1$.

4. Endemic Equilibrium Analysis

4.1. Endemic Gambling Equilibrium State of the Gambling System

The endemic gambling equilibrium represents a persistent steady state where gambling activity remains present in the community. At this equilibrium, at least one gambling-related compartment is strictly positive. That is, $E^* > 0,\ B^* > 0$ and $P^* > 0$.

The endemic steady state is represented as $E_1 = (S^*, E^*, B^*, P^*, D^*, R^*)$.

At equilibrium, set all the derivative of equation (1) to (6) to zero and express compartments in terms of $E^*$to give;

$$0 = \Lambda - \lambda S + \phi R - \mu S, \tag{22}$$

$$0 = \lambda S + \sigma R - (\alpha + \mu)E, \tag{23}$$

$$0 = \alpha E - (\delta + \gamma + \mu)B, \tag{24}$$

$$0 = \delta B - (\kappa + \rho + \mu)P, \tag{25}$$

$$0 = \rho P + (\eta + \mu)D, \tag{26}$$

$$0 = \gamma B + \kappa P + \eta D - (\phi + \sigma + \mu)R. \tag{27}$$

With force of gambling exposure a**s**

$$\lambda = (1 - C)\left(\omega + \beta \frac{B + P}{N}\right) \tag{28}$$

With $$N = S + E + B + P + D + R$$

From equation (24);

$$B^* = \frac{\alpha}{\delta + \gamma + \mu} E^* \tag{29}$$

From equation (25);

$$P^* = \frac{\delta}{(k + \rho + \mu)} B^* \tag{30}$$

Substitute equation (29) into (30),

$$P^* = \frac{\delta\alpha}{(\delta + \gamma + \mu)(\kappa + \rho + \mu)} E^* \tag{31}$$

From equation (26);

$$D^* = \frac{\rho P^*}{(\eta + \mu)} \tag{32}$$

Put (29) into (31) and simplifying to get,

$$D^* = \frac{\rho\delta\alpha}{(\delta + \gamma + \mu)(\kappa + \rho + \mu)(\eta + \mu)} E^* \tag{33}$$

From equation (27);

$$R^* = \frac{\gamma B^* + \kappa P^* + \eta D^*}{\phi + \sigma + \mu} \tag{34}$$

Substitute (29), (31) and (33) into equation (34) and simplifying further yields

$$R^* = \frac{\alpha E^*}{\phi + \sigma + \mu}\left[\frac{\gamma}{\delta + \gamma + \mu} + \frac{\kappa\delta}{(\delta + \gamma + \mu)(\kappa + \rho + \mu)} + \frac{\eta\rho\delta}{(\delta + \gamma + \mu)(\kappa + \rho + \mu)(\eta + \mu)}\right] \tag{35}$$

From equation (22);

$$0 = \Lambda - \lambda S^* + \phi R^* - \mu S^*$$

$$\Rightarrow S^* = \frac{\Lambda + \phi R^*}{\lambda^*} \tag{36}$$

Force of gambling exposure is givena**s**,

$$\lambda^* = (1 - C)\left(\omega + \beta \frac{B^* + P^*}{N^*}\right) \tag{37}$$

Adding equation (29) and (31) gives,

$$B^* + P^* = \frac{\alpha E^*}{\delta + \gamma + \mu}\left(1 + \frac{\delta}{\kappa + \rho + \mu}\right) \tag{38}$$

Put equation (37) into (38),

$$\lambda^* = (1 - C)\left(\omega + \frac{\beta\alpha E^*}{N^*(\delta + \gamma + \mu)}\left(1 + \frac{\delta}{\kappa + \rho + \mu}\right)\right) \tag{39}$$

From equation (23),

$$\lambda^* S^* + \sigma R^* = (\alpha + \mu)E^* \tag{40}$$

Substituting equation (35), (36) and (39) respectively into equation (40) will result to a scalar equation in E^*.

$$E^* = \chi(E^*)$$

A non-trivial solution $E^*> 0$ exists if and only if $R_e> 1$.

Thus, the endemic gambling equilibrium is $E_1 = (S^*, E^*, B^*, P^*, D^*, R^*)$ where each component is uniquely determined from E^*.

4.2. Global Stability of the Endemic Gambling Steady State

Theorem 4:

If the effective reproduction number is greater than one, the endemic equilibrium of the gambling system exhibits global asymptotic stability in Ω.

Proof:

Let the gambling model be defined on the positively invariant region $\Omega = \left\{ (S, E, B, P, D, R.) \in R_+^6 : N \le \frac{\Lambda}{\mu} \right\}$, For $R_e> 1$, the existence of an endemic gambling equilibrium is established and expressed as $E_1 = (S, E, B, P, D, R)$, $E^*> 0$, $B^*> 0$, $P^*> 0$.

Let the lyapunov function be,

$$\nu = \left(S - S^* - S^* In \frac{S}{S^*} \right) + \left(E - E^* - E^* In \frac{E}{E^*} \right) + \left(B - B^* - B^* In \frac{B}{B^*} \right) + \left(P - P^* - P^* In \frac{P}{P^*} \right) + \left(D - D^* - D^* In \frac{D}{D^*} \right) + \left(R - R^* - R^* In \frac{R}{R^*} \right). \quad (41)$$

With $\nu \ge 0$ for all states Ω and $\nu = 0$ if and only if the system is at E_1.

$$\frac{d\nu}{dt} = \left(1 - \frac{S}{S^*}\right)\frac{dS}{dt} + \left(1 - \frac{E}{E^*}\right)\frac{dE}{dt} + \left(1 - \frac{B}{B^*}\right)\frac{dB}{dt} + \left(1 - \frac{P}{P^*}\right)\frac{dP}{dt} + \left(1 - \frac{D}{D^*}\right)\frac{dD}{dt} + \left(1 - \frac{R}{R^*}\right)\frac{dR}{dt}. \quad (42)$$

Substituting equation (1) to (6) into (42) and simplifying using equilibrium identities will lead to,

$\frac{d\nu}{dt} \le 0$ with equality holding if and only if $S = S^*, E = E^*, B = B^*, P = P^*, D = D^*, R = R^*$. For all x> 0, $x - 1 - Inx \ge 0$, with equality if and only if x = 1.

Applying LaSalle's invariance principle. Let $M = \left\{ x \in \Omega : \frac{d\nu}{dt} = 0 \right\}$. The largest invariant subset of M is the singleton set $\{E_1\}$. Hence, $\lim_{t \to \infty} (S, E, B, P, D, R) = E_1$.

Therefore, the endemic gambling equilibrium E_1 is globally asymptotically stable in Ω whenever $R_e> 1$.

5. Sensitivity Analysis and Numerical Simulations

5.1. Sensitivity Assessment of the Effective Reproduction Number.

Sensitivity analysis is used to establish which parameter mainly influence the effective reproduction numberR_e, which governs whether gambling behaviour persists or dies out in the population. The normalized forward sensitivity index is adopted because it provides a dimensionless and policy-relevant measure of impact [5]; [17]. Let 'u' depend on a parameter 'p'. The normalized forward sensitivity index of 'u' with respect to 'p' is defined as

$$\gamma_u^p = \frac{\partial u}{\partial p} . \frac{p}{u}$$

This index quantifies the relative change in 'u' produced by a one-percent variation in 'p' [5]. Carrying out the sensitivity analysis on (19) using the parameters values on table 3 gives the sensitive indices of the respective parameters on table 4 with R_e = 1.458.

Table 3 Parameter Values of the Effective Reproduction Number

Parameters	Baseline Value	Source
β	0.62	[10].
C	0.35	[11].
α	0.40	[3]; [10].
δ	0.25	[13].
γ	0.15	[15]; [14].
κ	0.10	[14]; [1].
ρ	0.30	[10].
μ	0.02	[10]; [16].
Λ	20	Assumed value.
ω	0.10	Assumed value.
ϕ	0.05	Assumed value.
σ	0.08	Assumed value.
η	0.12	Assumed value.

Table 4 Result of Sensitivity Index on Parameter

Parameters	Sensitivity index
β	+1.000
C	-0.538
α	+0.048
δ	+0.102
γ	-0.214
κ	-0.173

ρ	+0.091
μ	-0.067

From table 4, the normalized forward sensitivity analysis reveals that the exposure rate (β) has the highest positive sensitivity, indicating that gambling persistence is most responsive to digital betting exposure. Policy enforcement (C) shows the strongest negative influence, confirming that regulatory interventions are highly effective in reducing gambling spread. Recovery-related parameters (γ, κ) also significantly suppress R_e, while debt accumulation (ρ) contributes positively to gambling persistence. The numerical sensitivity graph plots showing the effect of each parameter on the effective reproduction number are shown in figure 2 to figure 9. Each plot shows how R_e changes when the parameter is varied from 0.5× to 1.5× its baseline value, with a dashed line at $R_e = 1$ (elimination threshold), and a dotted vertical line at 1× baseline. Baseline value used in all plots is $R_e \approx 1.458$.

5.2. Numerical Simulation of the Endemic Gambling Equilibrium.

MATLAB software was used to run the endemic equilibrium numerical simulations of the extended youth gambling model using thebaseline parameters in table 3. The simulation produces a stable endemic trajectory (all compartments settle to positive steady levels), consistent with an endemic gambling equilibrium. Using the R_e expression (24) previously derived, the computed value is 1.458 making use of the corresponding parameter values in table 3. Endemic equilibrium from simulation (approximate steady state) are (S^*, E^*, B^*, P^*, D^*, R^*) ≈ (212.12, 130.24, 124.03, 73.82, 158.13, 299.65). I approximated the endemic equilibrium by taking the solution at the final simulation time (t = 200), so that the endemic gambling burden (active + problem gamblers) is ($B^* + P^*$) ≈ 124.03 + 73.82 ≈ 197.85. The simulation graphs are shown in figure 10 to 13.

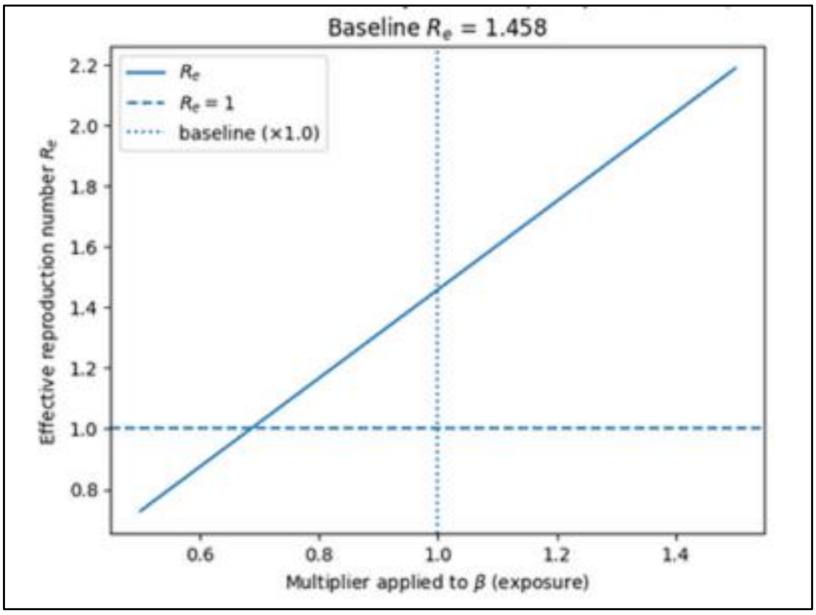

Figure 2 Graphical effect of β on R_e.

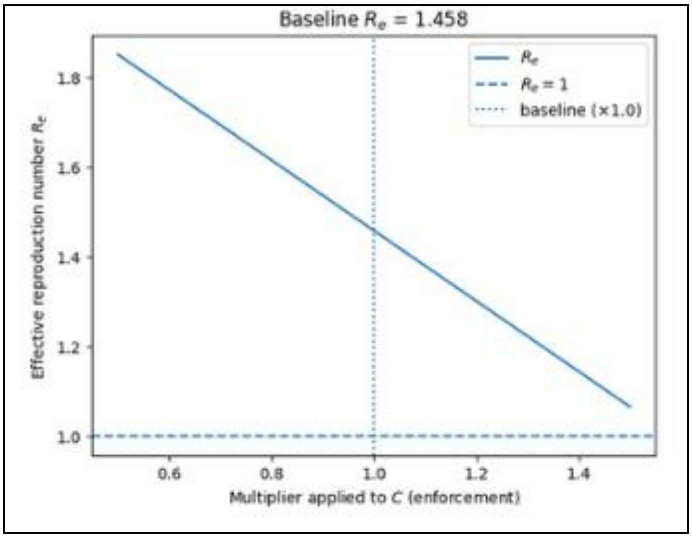

Figure 3 Graphical effect of C on R_e.

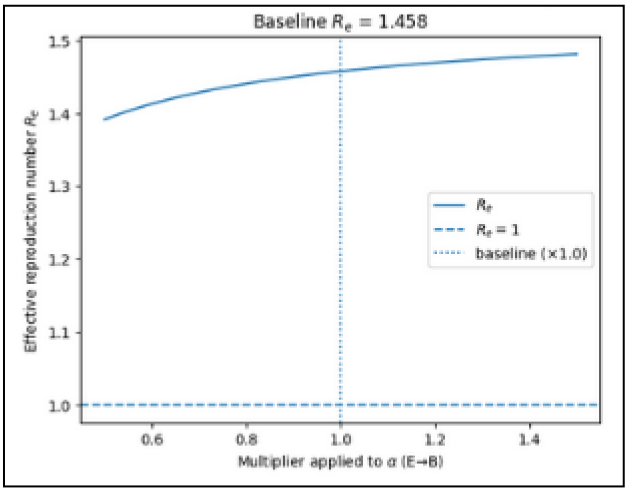

Figure 4 Graphical effect of α on R_e.

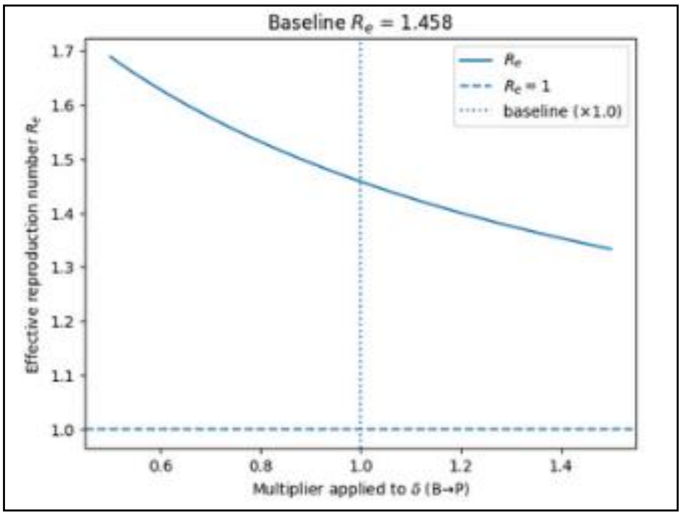

Figure 5 Graphical effect of δ on R_e.

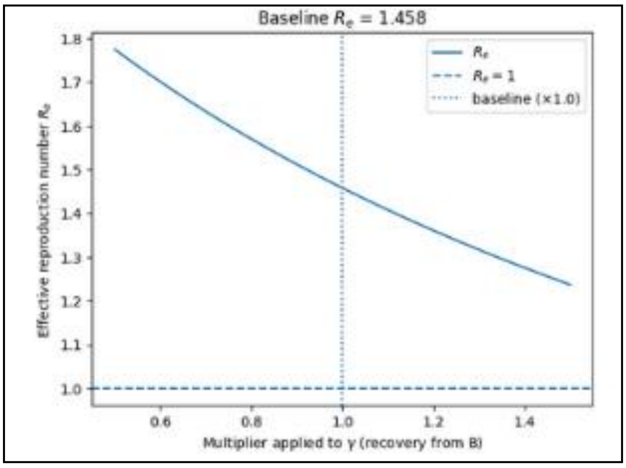

Figure 6 Graphical effect of γ on R_e.

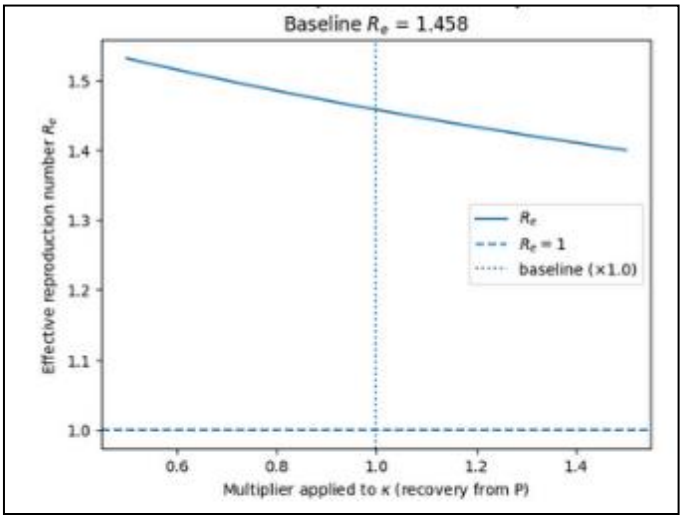

Figure 7 Graphical effect of κ on R_e.

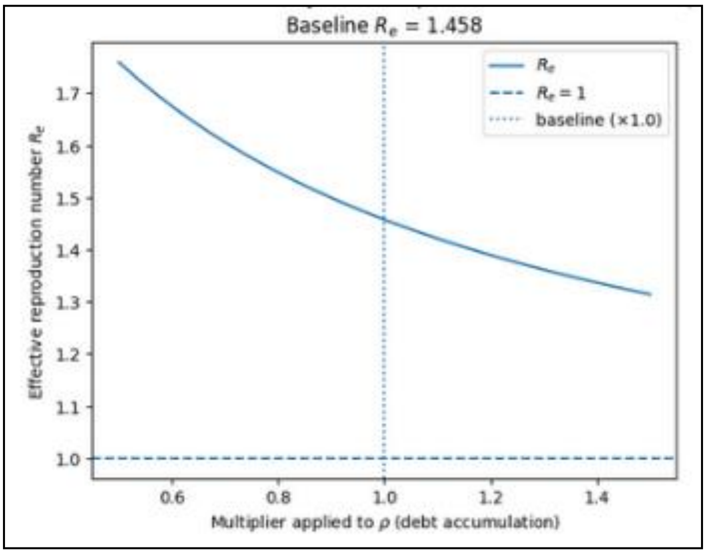

Figure 8 Graphical effect of ρ on R_e.

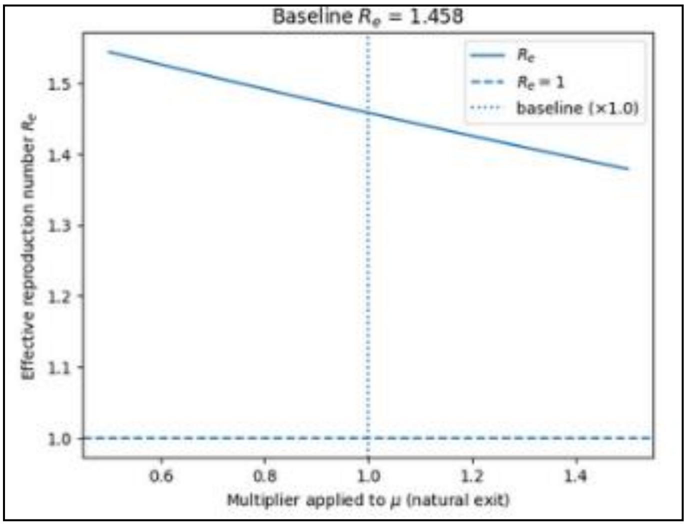

Figure 9 Graphical effect of μ on R_e.

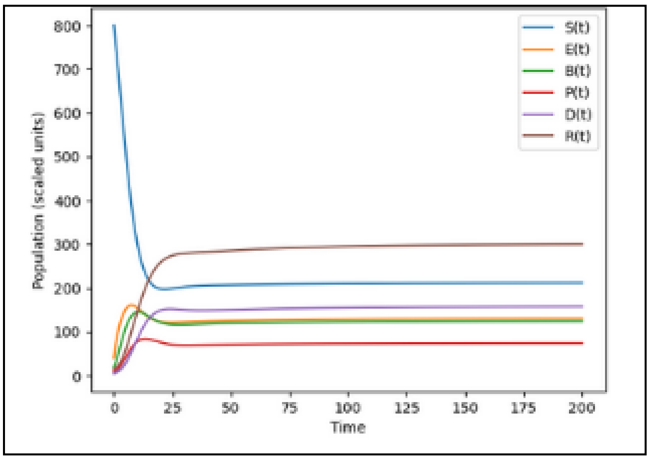

Figure 10 Graph of the youth gambling of all the state variables.

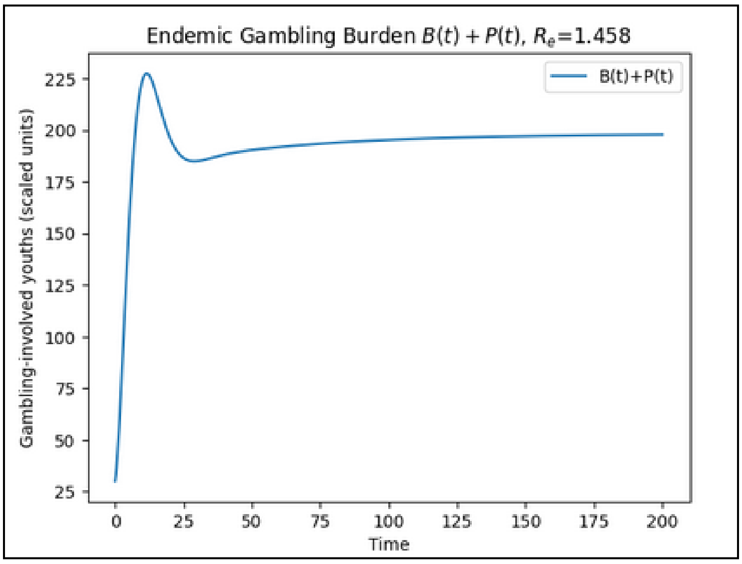

Figure 11 Graph of the endemic gambling burden B(t) + P(t).

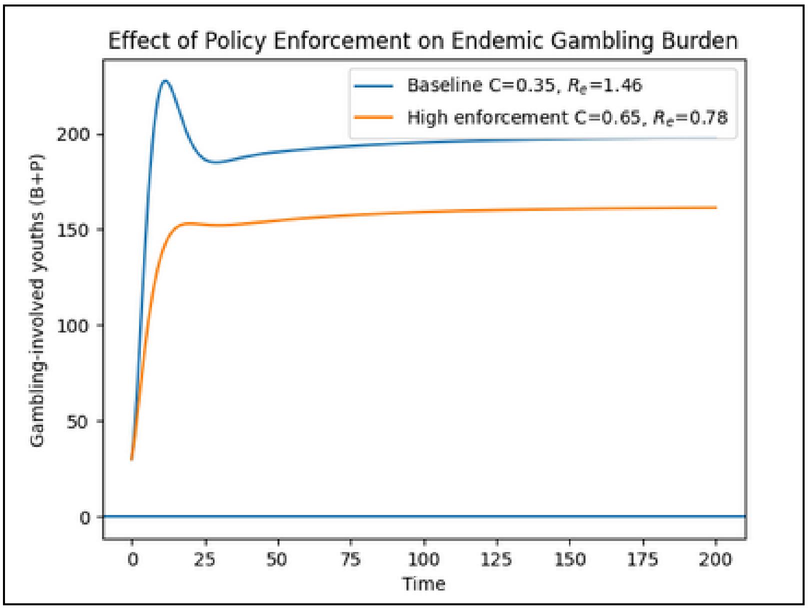

Figure 12 Graph of the effect policy enforcement on endemic gambling burden.

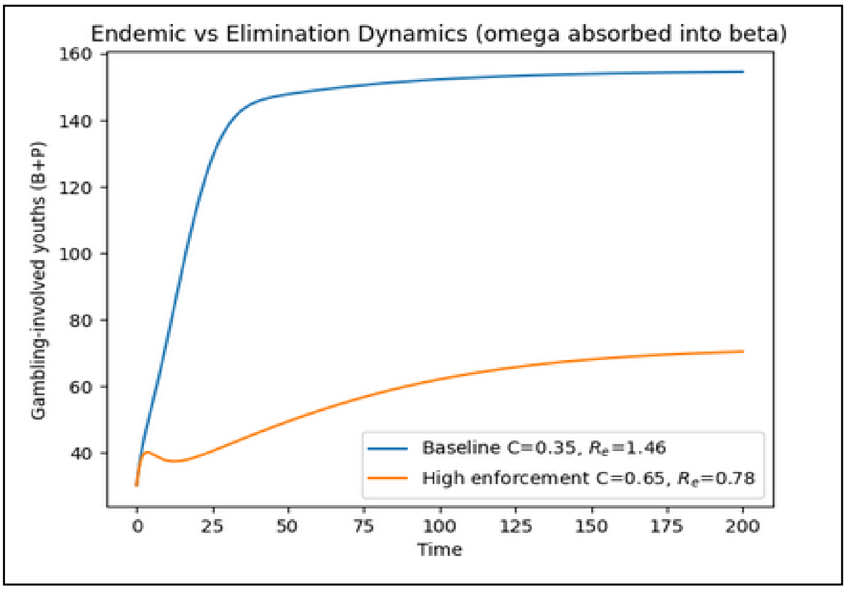

Figure 13 Graph of endemic vs youths gambling (B+P) elimination dynamics.

6. Discussions

6.1. Effect of β on R_e (betting exposure/digital–peer influence)

- Figure 2 shows: R_e increases almost linearly as β increases.
- Reason: β appears as a multiplicative factor in R_e, thus proportional change in β gives proportional change in R_e.
- Threshold implication: Reducing β can push R_e toward 1 and below depending on how large the reduction is.
- Policy meaning: This confirms that controlling digital marketing reach, promotions, and peer-driven betting influences is the most direct lever for reducing gambling persistence.

6.2. Effect of C on R_e (policy enforcement strength)

- Figure 3 shows: R_e decreases almost linearly as enforcement C increases.
- Reason: R_e is proportional to $(1-C)$. Stronger enforcement reduces the effective “contact” intensity.

- Threshold implication: Increasing C moves the system toward elimination (toward $R_e < 1$).
- Policy meaning: Strengthening Nigeria Lottery Regulatory Commission (NLRC) enforcement (ad restrictions, age verification, self-exclusion compliance and penalties) has a large impact. This plot visually supports the model claim that regulation is a high-payoff control strategy.

6.3. Effect of α on R_e (transition from exposure to betting)

- Figure 4 shows: R_e increases with α but slowly and with diminishing returns (curve flattens).
- Reason: α appears in the fraction $\frac{\alpha}{\alpha + \mu}$; as α becomes large, α + μ grows similarly, so the ratio saturates.
- Threshold implication: Changing α alone is unlikely to drive R_e below 1 unless combined with other interventions.
- Policy meaning: Education and behavior-change programs that delay first-time betting help, but their impact is less dominant than direct exposure control.

6.4. Effect of δ on R_e (escalation from betting to problem gambling)

- Figure 5 shows: R_e decreases as δ increases (a downward nonlinear curve). This means that in R_e expression, δ appears both positively inside $\left(1 + \frac{\delta}{\kappa + \rho + \mu}\right)$ and negatively through the term (δ + γ + μ) in the denominator.

With the baseline values in table 3, the denominator effect dominates, so increasing δ shortens time in B enough that overall "transmission" drops.

Interpretation caution: This means that, under this formulation, faster movement from B to P reduces the time

spent in the main "spreading" class B.

6.5. Effect of γ on R_e (recovery from betting)

- Figure 6 shows: R_e decreases strongly as γ increases (downward trend).
- Reason: γ increases the removal rate from B via (δ + γ + μ), reducing the average time an individual remains an active bettor.
- Threshold implication: Increasing recovery support can push Re toward 1, especially when combined with reductions in β.
- Policy meaning: Awareness campaigns, counseling access, and early "brief interventions" for active bettors are highly effective.

6.6. Effect of κ on R_e (recovery from problem gambling)

- Figure 7 shows: R_e decreases as κ increases, but the slope is weaker compare to γ.
- Reason: κ acts on the problem gambling compartment P through (κ + ρ + μ). It reduces persistence among P, which contributes indirectly to new exposure.
- Policy meaning: Specialized treatment (rehabilitation, addiction therapy) matters, but it is usually more resource-intensive and affects a smaller group than γ-type early interventions.

6.7. Effect of ρ on R_e (debt accumulation)

- Figure 8 shows: R_e decreases with ρ (downward curve). Increasing ρ reduces the fraction, lowering R_e, since ρ appears in (κ + ρ + μ) in the denominator of the term $\frac{\delta}{\kappa + \rho + \mu}$.
- Interpretation caution: It affects the P-duration term but does not create extra "infectiousness" directly.

6.8. Effect of μ on R_e (natural exit rate)

- Figure 9 shows: R_e decreases gently as μ increases.

- Reason: μ increases departure from all key states and appears in multiple denominators, shortening average time in exposed/gambling states.
- Policy meaning: It confirms the model's internal consistency. Faster exit reduces persistence.

6.9. Explanation of the Endemic Equilibrium Simulation Graph (figure 10 to 13)

The endemic equilibrium simulation graph illustrates the long-term dynamics of youth gambling behaviour under baseline parameter values. Starting from mixed initial conditions, all state variables evolve over time and converge to constant positive levels, confirming the existence of a stable endemic gambling equilibrium.

In the early phase of the simulation, the susceptible population declines rapidly as digitally exposed individuals transition into gambling due to high exposure intensity and peer influence. This is accompanied by a corresponding rise in the exposed and active gambling compartments. As time progresses, active gamblers increasingly transition into problem gambling and debt accumulation, leading to sustained growth in the problem gambling and indebted classes.

The recovery compartment initially increases due to behavioural change and intervention, but stabilizes at a positive level rather than eliminating gambling entirely. This reflects the effect of relapse driven by continued digital exposure and imperfect policy enforcement. Consequently, recovery alone is insufficient to drive gambling elimination when the effective reproduction number exceeds unity.

The combined gambling burden, represented by the sum of active and problem gamblers, approaches a steady value over time. This convergence demonstrates that gambling persists at a stable endemic level rather than exhibiting unbounded growth or extinction. The absence of oscillations indicates that the system settles into a stable equilibrium rather than cyclical behaviour.

Overall, the simulation confirms the analytical results: when the effective reproduction number is greater than one, youth gambling persists in the population despite recovery mechanisms. The endemic equilibrium reflects a balance between gambling initiation, escalation, recovery, relapse, and policy enforcement, highlighting the need for structural interventions that reduce exposure and strengthen regulatory control to shift the system toward elimination.

7. Conclusion

This study developed and analyzed a novel mathematical model to examine youth gambling dynamics in Nigeria by explicitly incorporating digital exposure, gambling progression, financial debt accumulation, recovery with relapse, and policy enforcement mechanisms. The model was shown to be mathematically well-posed, with positive and bounded solutions, and to admit both gambling-free and endemic equilibria governed by a clearly defined threshold quantity. Analytical results demonstrated that the effective reproduction number serves as a decisive indicator of whether gambling behavior will fade out or persist within the youth population.

Sensitivity analysis revealed that gambling persistence is most strongly driven by digital and peer-based exposure, while regulatory enforcement exerts a powerful suppressive effect. Recovery-related processes reduce gambling transmission but are insufficient on their own to eliminate gambling when exposure remains high. Numerical simulations calibrated with Nigeria-based parameter values confirmed the existence of a stable endemic gambling equilibrium under current conditions, characterized by sustained levels of active and problem gambling and significant accumulation of gambling-related debt.

Overall, the model provides quantitative evidence that youth gambling in Nigeria is not merely an individual behavioral issue but a self-reinforcing social process sustained by digital access, weak enforcement, and economic vulnerability. Without structural intervention, the system naturally evolves toward persistent gambling prevalence rather than spontaneous elimination.

7.1. Recommendations

Regulatory enforcement of gambling laws should be strengthened, particularly in controlling digital advertising and enforcing age restrictions, as exposure is the primary driver of gambling persistence. Targeted youth education programs should focus on preventing initial gambling engagement rather than only addressing addiction at later stages. Accessible early-intervention and recovery services should be expanded to reduce progression into problem gambling. Finally, financial literacy and debt-management initiatives should be integrated into gambling harm reduction strategies to weaken the feedback between debt and continued gambling behaviour.

7.2. Future work

Future studies may extend the present model by incorporating optimal control strategies to formally evaluate the cost-effectiveness of regulatory enforcement, public awareness campaigns, and treatment interventions. Stochastic versions of the model could be developed to capture uncertainty in youth behavior and fluctuations in digital exposure. Further extensions may include age-structured or gender-specific dynamics to reflect heterogeneity in gambling behavior across subpopulations. Integrating real-time administrative data from regulatory agencies would improve parameter estimation and predictive accuracy. Finally, coupling the gambling model with economic or mental-health outcome models would allow a more comprehensive assessment of the long-term societal impact of youth gambling in Nigeria.

Compliance with ethical standards

Acknowledgments

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Disclosure of conflict of interest

We the authors declare that there is no conflict of interest regarding the publication of this paper.

Statement of ethical approval

This study utilized secondary data obtained from publicly available sources to inform the mathematical modeling of youth gambling behavior in Nigeria. No human or animal subjects were directly involved and ethical approval was not required.

Statement of informed consent

Informed consent was not required as the study was based exclusively on publicly available and anonymized secondary data.

References

[1] Abbott, M. (2017). Gambling and gambling harm in New Zealand: A 28-year case study. International Journal of Mental Health and Addiction, 15(6), 1221–1241. https://doi.org/10.1007/s11469-017-9767-5

[2] Adebisi, Y. A., Alaran, A. J., Akinokun, R. T., Micheal, O., Ilesanmi, E. B., & Lucero-Prisno, D. E. (2021). Gambling addiction among youths: A public health concern in Nigeria. Annals of Global Health, 87(1), 1–6. https://doi.org/10.5334/aogh.3253

[3] Akanbi, O. A., Lawal, O. A., & Sadiq, T. A. (2023). Youth gambling and socioeconomic vulnerability in Nigeria. African Journal of Economic and Management Studies, 14(2), 215–230. https://doi.org/10.1108/AJEMS-09-2022-0441

[4] Castillo-Chavez, C., Feng, Z., & Huang, W. (2002). On the computation of R_0 and its role on global stability. In C. Castillo-Chavez et al. (Eds.), Mathematical approaches for emerging and reemerging infectious diseases (pp. 229–250). Springer.

[5] Chitnis, N., Hyman, J. M., & Cushing, J. M. (2008). Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model. 70(5), 1272–1296. https://doi.org/10.1007/s11538-008-9299-0

[6] Do, T. H., & Lee, Y. (2014). Modeling the dynamics of gambling addiction. Applied Mathematics and Computation, 234, 352–363. https://doi.org/10.1016/j.amc.2014.02.057

[7] Gainsbury, S. (2015). Online gambling addiction: The relationship between internet gambling and disordered gambling. Current Addiction Reports, 2(2), 185–193. https://doi.org/10.1007/s40429-015-0057-8

[8] Hethcote, H. W. (2000). The mathematics of infectious diseases. SIAM Review, 42(4), 599–653. https://doi.org/10.1137/S0036144500371907

[9] Kosgei, M. K., & Kibet, K. C. (2025). Mathematical modeling of gambling addiction dynamics with relapse. Journal of Applied Mathematics, 2025, Article 889214. https://doi.org/10.1155/2025/889214

[10] National Bureau of Statistics. (2021). Youth employment and digital economy report. NBS.

[11] Nigeria Lottery Regulatory Commission. (2022). Annual regulatory performance report. NLRC.

[12] Njemanze, V. C., Nwosu, O. E., & Njemanze, C. I. (2020). Online gambling behavior and youth risk exposure in Nigeria. Journal of Behavioral Addictions, 9(4), 1021–1030. https://doi.org/10.1556/2006.2020.00082

[13] Okorie, N., & Adebayo, E. (2020). Digital advertising and gambling participation among Nigerian youths. Journal of African Media Studies, 12(3), 395–410. https://doi.org/10.1386/jams_00028_1

[14] Orji, R., Tondello, G. F., & Nacke, L. E. (2022). Behavioral intervention strategies for reducing online gambling addiction. Computers in Human Behavior, 128, 107123. https://doi.org/10.1016/j.chb.2021.107123

[15] Shaffer, H. J., LaPlante, D. A., & Nelson, S. E. (2014). The epidemiology of pathological gambling. Psychiatric Clinics of North America, 37(3), 345–357. https://doi.org/10.1016/j.psc.2014.05.002

[16] United Nations Department of Economic and Social Affairs. (2020). World population prospects 2019. UNDESA.

[17] van den Driessche, P., & Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Mathematical Biosciences, 180(1–2), 29–48. https://doi.org/10.1016/S0025-5564(02)00108-6