International Journal of Science and Research Archive

eISSN: 2582-8185

Cross Ref DOI: 10.30574/ijsra

Journal homepage: https://ijsra.net/

(Review Article)

Multivariate GARCH Models for Portfolio Risk Management: A Comparative Study

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International Journal of Science and Research Archive, 2026, 18(02), 492-505

Publication history: Received on 06 January 2026; revised on 12 February 2026; accepted on 14 February 2026

Article DOI: https://doi.org/10.30574/ijsra.2026.18.2.0282

Abstract

The research article presents an overall comparative study of multivariate GARCH M-GARCH in portfolio risk management, where three prevailing specifications VEC, CCC and DCC models are considered. We take daily closing prices of four major assets of the year 2018 through to 2023; S&P 500, NASDAQ-100, gold futures, and US Treasury Bonds, to estimate conditional variances, covariances, and dynamic correlations using maximum likelihood estimation. The descriptive statistics indicate that volatility is highly concentrated in clustering and time varying across assets with NASDAQ having the highest volatility (2.08) and significant negative skewness that shows non-normal returns. Comparison on models based on information criteria indicates that the Dynamic Conditional Correlation (DCC) specification has better performance with less computation need, fewer 8 parameters as compared to 21 (VEC) and greater log-likelihood with high improvement (164.67 units). Adequate model specification is shown through diagnostic testing using Ljung-Box test and ARCH-LM tests. Empirical results indicate that the volatility persistence (alpha + - 0 = 0.98) and the dynamics of high correlation (beta = 0.9321) are high which indicates long-memory properties and mean-reverting behavior which does not support constant correlation assumptions.

Keywords: Multivariate GARCH; Dynamic Conditional Correlation; Portfolio Risk Management; Volatility Clustering; Value-at-Risk

1. Introduction

Portfolio risk management is a core issue in modern financial practice, and there is a need to have a precise quantification of the volatility of the individual assets and the relationship between the portfolio constituents. The traditional methods, which are based on constant correlation assumptions, have been shown to be highly inadequate in reflecting dynamics in financial market relations, especially when market is under stress and correlations are often enhanced significantly (Marti et al., 2021). The original contribution of Engle (1982) is the univariate ARCH models which was later generalized based on the generalized ARCH (GARCH) model by Bollerslev (1986) (Shephard, 2020). Since the introduction, multivariate GARCH models have become extremely prominent in both the literature and practice. The VEC specification was first proposed by Bollerslev, Engle, and Wooldridge, then the constant conditional correlation (CCC) specification of Bollerslev, and then the dynamic conditional correlation (DCC) specification of Engle (Ballestra et al., 2025).

The main goal of the research is the systematic comparative analysis of three principal M-GARCH specifications, namely VEC, CCC, and DCC, in the scope of portfolio risk management. Using modern financial data of various asset types, we examine how well our models perform in various metrics that include goodness-of-fit, efficiency of computation, as well as the feasibility of their practical implementation. The question in secondary research is how significant the dynamic correlation modeling in portfolio optimization and Value-at-Risk computation is versus constant correlation methodology.

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1.1. Research Objectives

- To conduct a comprehensive comparative evaluation of VEC, CCC, and DCC M-GARCH models in terms of their theoretical foundations, structural characteristics, and estimation procedures.
- To assess the empirical performance of these three model specifications using contemporary financial data across multiple asset classes, with particular emphasis on volatility forecasting accuracy and correlation dynamics.
- To examine the computational efficiency and practical implementability of each model specification, considering the trade-offs between model complexity and estimation feasibility.
- To investigate the impact of dynamic versus constant correlation assumptions on portfolio optimization outcomes and risk metric calculations, particularly Value-at-Risk (VaR) estimations.
- To provide evidence-based recommendations for practitioners regarding the selection of appropriate M-GARCH specifications under different market conditions and portfolio compositions.

1.2. Research Questions

To address the stated objectives, this study is guided by the following research questions:

Primary Research Questions:

- How do VEC, CCC, and DCC M-GARCH models compare in their ability to capture time-varying volatility and correlation dynamics in multi-asset portfolios?
- Which model specification provides superior out-of-sample forecasting performance for portfolio risk metrics across different market regimes?

Secondary Research Questions:

- What is the magnitude of improvement in risk measurement accuracy when employing dynamic correlation models (DCC) compared to constant correlation models (CCC)?
- How do computational requirements and parameter estimation stability differ across the three M-GARCH specifications, and what are the implications for real-world portfolio management applications?
- Under what market conditions and portfolio characteristics does each model specification demonstrate optimal performance?
- To what extent do the correlation dynamics captured by DCC models translate into economically significant differences in portfolio allocation decisions and risk-adjusted returns?

1.3. Significance of the Study

The study has an impact on the academic literature and financial practice in a number of significant aspects.

1.3.1. Theoretical Contributions

Theoretically, the study builds on the current literature in the field of multivariate volatility modeling by offering a single comparison framework of comparing competing M-GARCH models. Although single studies have investigated a particular model in isolation, there are few indications of comprehensive comparative studies that compare VEC, CCC, and DCC models on a systematic basis of appraising data presented and appraisal standards. This paper fills this gap by providing information on the comparative advantages and disadvantages of each method, hence enhancing our knowledge about the concept of conditional heteroskedasticity under multivariate.

1.3.2. Practical Contributions

In the case of practitioners, this research has a number of insights that are useful. First, it offers empirical data on the practical trade-offs between the complexity of the model and forecast accuracy that would allow portfolio managers to make informed decisions in choosing risk models. Second, providing economic importance of dynamic correlation modeling in the optimization of portfolios and the calculation of VaR, the paper aids in justification of extra costs incurred in the computation of more specifications. Third, the determination of market conditions that each model is best suited in provides practical advice to the choice of models in various investing situations.

1.3.3. Methodological Contributions

This study is methodologically useful as it introduces a widely encompassing assessment scheme where numerous dimensions of performance are introduced at once- such as statistical fitness, predictive accuracy, computational effectiveness and economic importance. This comprehensive method is a more detailed measure than the ones that concentrate on individual measures of performance and sets a precedent in future comparative research of the field of financial econometrics.

1.3.4. Timeliness and Relevance

The current market forces that are highly volatile, changing the correlation aspect quickly at stressful times, and an increasing regulatory focus on sound risk management practices, enhance the relevance of the study. Due to the increased capital adequacy regulations imposed on financial institutions by regulatory frameworks such as Basel III, the precise quantification of portfolio risk has not only become an academic interest, but also a regulatory necessity. The practical significance of information on the most appropriate methods of modeling risk trends is thus urgent.

Moreover, as algorithmic trading, high frequency data and complex financial instruments have become popular, the requirement of sophisticated risk models that can nonetheless be realized has never been more. The comparative analysis of M-GARCH models presented in this study is directly related to this objective since it provides an evaluation of models that strike the optimal balance between theory and its application.

2. Literature Review

2.1. Theoretical Foundations of GARCH Models

The ARCH models were developed and presented by Engle (1982) as a breakthrough in the financial econometrics, as these models allow investigators to express conditional variance as a dependence on previous squared innovations (Aser, 2023). This invention was especially useful in financial applications, where heteroskedasticity, or the tendency of volatility, is widespread in the empirical data. Financial econometrics before formulating ARCH, the assumption of variance constancy (homoskedasticity) was made, which is empirically invalid as can be seen through volatility clustering with quiet periods being replaced by periods of volatility, then returning us to relative calm. The univariate ARCH(q) model assumes reliance of conditional variance on q lagged squared residuals, which is expressed as:

$$\sigma^2{}_t = \omega + \Sigma_{i=1}{}^g \alpha_i \varepsilon^2{}_{t-i}$$

in which ω is the constant term, α ı coefficients reflect the effects of shocks, and ε 2 10 -1: -1 lagged squared residuals. This model is able to allow a response of variance to market surprises, which can be empirically seen in financial data. The original work by Engle (1982), which received a later Nobel (2003) award, highlights the basic significance of the framework to financial economics (Krauss, 2024).

An extension of the ARCH done by Bollerslev (1986) added generalized ARCH framework which allowed a more parsimonious parametrization by adding lagged conditional terms of variance (Ghalanos, 2020). The empirical behavior of the GARCH(1,1) specification, although it is simple, is astonishingly good in a variety of financial applications:

$$\sigma^2{}_t = \omega + \alpha\varepsilon^2{}_{t-1} + \beta\sigma^2{}_{t-1}$$

where 8 is a volatility persistence coefficient- the degree to which recent volatility is determined by past volatility. As alpha + beta becomes closer to one, volatility itself has permanent shock effects, which suggest that volatility surprises have permanent effects on quantification of conditional risks. This sparse form coupled with the interpretability of the parameters and computational tractability have made GARCH(1,1) an industry standard in the risk management use case, which is used by many financial institutions, central banks and regulators across the globe.

2.2. Multivariate GARCH Specifications

The move beyond univariate to multivariate models required special focus on dimensionality limitations because unrestricted VEC models are burdensome in terms of the parameters to estimate. In the case of systems with N assets, the unrestricted variance-covariance matrices have N(N+1)/2 elements that need to be estimated. Parameters are highly exaggerated by GARCH extensions: univariate GARCH(1,1) has 3 parameters per asset; extensions are not limited in multivariate GARCHs, and this scales up and up. It is evident that even the simplest four-asset portfolios face very extreme estimation problems with unconstrained specifications.

The VEC framework was proposed by Bollerslev, Engle, and Wooldridge (1988), which was the first to explicitly parameterize the conditional variance-covariance matrix (H 0) by:

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'H_{t-1}B$$

and where C is made of N N lower triangular matrix (constant terms), A and B are N N parameter matrices of shock transmission and persistence. These vectorial specifications allow interdependencies to be modeled in full: it is explicitly parametrized in terms of covariances of shocks between assets (through A matrix) and dynamic covariance persistence (through B matrix). Theoretically exhaustive, the computational requirements present VEC with a maximum limited practical capacity of at most two or three assets in a system. The computational costs of parameter estimation grow exponentially with asset count, maximum likelihood optimization turns out to be computationally infeasible and difficulty of convergence is encountered routinely.

The CCC model by Bollerslev (1990), in its turn, is a fine compromise between the theoretical comprehensiveness and computational tractability. The specification also conjectures this restriction that conditional correlations are fixed in the long run, but conditional variances are dynamically changing:

$$H_t = D_tRD_t$$

and D ℕ diagonal matrix of conditional standard deviations (σ i N) of each asset, and R is constant correlation matrix. This decomposition is also elegant in the sense that it does not require estimation of N N covariance matrix elements: instead, N GARCH(1,1) specifications, univariate (3N) parameters, and N (N-1)/2 correlations are estimated only once. CCC needs only 18 parameters compared to 21 to specify unrestricted VECs with four-asset portfolios, and this increases exponentially with the size of the portfolio.

The critical assumption of the CCC specification, which is that correlations are constant, seems to be weak in the face of empirical evidence. Nevertheless, Bollerslev (1990) proves that CCC works surprisingly well in a wide range of empirical applications, implying that either correlations vary around their means or these means can be constant over a long period of time. However, there is a significant evidence that records change in correlation over time, especially during market stress periods where there is a significant change in correlation. The empirical violation of constant correlation assumptions that are visible in the phenomenon of the flight-to-quality.

The DCC model of the article by Engle is another significant step in the right direction, with conditional correlations being modeled as time-varying processes, which allows capturing the dynamics of correlations and at the same time is computationally tractable (Xue, 2023). The DCC specification is parsimonious because it employs the two-step estimation procedure whereby conditional variances are estimated using univariate GARCH specifications, followed by estimating conditional correlations using standardized residuals:

$$H_t = D_tR_tD_t$$

where R 0 changes dynamically due to:

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha Z_{t-1}Z_{t-1}' + \beta Q_{t-1}$$

and R = Q 0 -1Q 0 Q 0 -1, where Z 0 is a standardized residual and Q 0 is a diagonal matrix of 1/sqrtQ 0 diagonal elements. This decomposition is computationally efficient (needs only 2 extra parameters, 0 0) and empirically flexible (allowing correlation dynamics to react to current shocks, 0 0) and its flexibility allows the exploration of correlation dynamics that respond to current shocks (0 0) and past levels of correlation (0).

2.3. Empirical Applications and Comparative Studies

Various studies have recorded excellent performance of time-varying correlation models compared with constant correlation specifications in the portfolio management setup. Leveraging high-frequency data on a variety of equity markets, Perez Riaza & Gnabo, (2024) reveal that the DCC models make the Value-at-Risk forecasting performance significantly higher, especially at turbulent market times when correlations heighten considerably. Their backtesting shows that dynamic correlation models predict hits that are near to theoretical forecasts where constant correlation models systematically underestimate tail risk that occurs during stress events.

Decker & Ferla, (2023) compare performance of portfolio optimization between CCC and DCC specifications with the traditional Markowitz strategies of using unconditional correlations. Their study uses daily data in portfolios of 5-15 equity assets and they found out that dynamic correlation models create portfolios with much better risk-return tradeoffs compared to the same model with static specifications.

Asymmetric DCC specifications drafted by Davidescu et al., (2025) are able to record variations in correlations between positive and negative shocks and prove to be specifically relevant to market stress events. Their formulation of ADCC allows correlations to respond more significantly to negative equity returns than it is to positive returns based on empirical findings that bad news leads to greater interdependence compared to good news.

The article by Costa, (2020) offers extensive literature review of multivariate GARCH models up to 2008, and reports a phenomenal rise in theoretical extensions and application. They classify models by the methods of parametrization and comment on the pros and cons of the computations. The authors point out that although theoretical extensions continue to grow, in practice, either CCC or DCC specifications are used, reflecting the good trade-off between flexibility and tractability of these models.

Recent implementations in cryptocurrency markets and energy commodities reveal that M-GARCH still has a place in new asset classes (Husain, 2024). A study of cryptocurrency shows that correlation persistence is extremely high, by far exceeding the correlation of traditional assets, implying that digital assets are highly-herding and do not enjoy much diversification advantage.

2.4. Portfolio Risk Management Applications

The current stress in modern portfolio management is on quantification of tail risk and dynamic hedging instead of mean-variance optimization. VaR and Conditional Value-at-Risk (CVaR) measures are measures of possible extreme losses, which is essential in the calculation of regulatory capital under Basel III frameworks. Zouari, (2022) suggests that the conditional correlation forecasting is essential in correct estimation of VaR because portfolio tail risk are concentrated when there is coincidental increase in asset correlations in conjunction with higher volatility. The conventional constant correlation models believe that diversification benefits would occur during the crisis when they are actually lost and that extreme tail risk is systematically underestimated.

Ngo, (2022) come up with multivariate extension of intensity-based extreme value theory alongside GARCH specifications, which allows them to better estimate tail risk. Their strategy is accommodating the conditions of correlation as well as extreme dependence structure modifications in the stress episodes. Applied to currency portfolio indicates that constant correlation approaches understate 99% confidence Value-at-Risk by a factor of about 35, and understate it significantly on even more extreme quantiles (Hertrich, 2025). The implication of this finding is far reaching in terms of regulatory capital adequacy because regulators are progressing toward placing greater focus on extreme quantile measures of risk.

Malandreniotis, (2024) discusses the value of correlation forecasts in the portfolio management using the utility maximization model. His discussion shows that correlation prediction gains will result in valuable utility increases or over multiple basis points of the returns to their portfolio each year, to risk-averse investors.

The seminal work of Saliya, (2025) has shown the correlation in the equity markets to in fact skyrocket when extreme negative returns are experienced, which in effect contravenes the assumptions of constant correlation at its very core. His evidence on the developed and the emerging markets indicates that correlation in the worst 5% return observations is 0.67, whereas it is 0.33 in the entire sample.

2.5. Regulatory and Practical Implementation Considerations

Interpretations of Basel III regulatory frameworks focus on Value-at-Risk and stressed Value-at-Risk calculations that need proper conditional volatility and correlation estimates. When financial institutions apply these frameworks, the volatility models adopted have to be a balance between theoretical advancement and operational obtainability and regulatory approval. The GARCH models, especially DCC specifications have achieved wide regulatory acceptance with numerous central banks and financial supervisors specifically suggesting their use.

The Lynch et al., (2023) records that JPMorgan, Goldman Sachs and key central banks use multivariate GARCH specification in their risk management system.

The most recent literature focuses on the hybrid methods of implementing GARCH specifications with machine learning methods. Subsequent extensions, such as those by Pan et al., (2024), use neural networks that predict conditional correlations in terms of observable variables in the market.

3. Methodology

3.1. Data and Sample Characteristics

The present investigation makes use of the everyday closing values of four significant financial resources between January 1, 2018, and December 31, 2023, obtained on Yahoo Finance. The portfolio of the assets includes S&P 500 Index (SPX), NASDAQ-100 Index (NDX), gold futures (GC), and the US Treasury Bond Index (TLT). This choice has created a sufficient diversification in terms of equity (large-cap and technology segment), commodities, and fixed income.

The computations of daily returns are calculated as: r 0 = [ln(P 0) - ln(P 0 1) × 100], which return us with returns in percentage, which are expressed in percentage terms. Such transformation supports the interpretation and comparison of different asset classes. The sample size is 1,507 trading observations, which offers significant levels of freedom to estimate the parameters and make a statistical conclusion.

3.2. Model Specifications

VEC Model: The conditional variance-covariance matrix (H 0) is parameterized directly in the form of the vector error correction specification:

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon_{t-1}'A + B'H_{t-1}B$$

C, A and B are parameter matrices that need to be estimated. Though one would consider it to be all-inclusive, the VEC specification adds 21 parameters to the four-asset system, which makes it difficult to estimate.

CCC Model: The constant conditional correlation specification breaks down H 0 as:

$$H_t = D_t R D_t$$

Here, D O includes diagonal conditional standard deviations (S I O). This breakdown minimizes parameters yet volatility dynamics are preserved.

DCC Model: The dynamic conditional correlation specification is an extension of the CCC framework:

$$H_t = D_t R_t D_t$$

Where R 0 is a dynamically changing quantity:

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha Z_{t-1}Z_{t-1}' + \beta Q_{t-1}$$

And R = Q -1QQQ -1 with Q representing the diagonal matrix of the Q elements of Q. This specification allows the flexibility in modelling the correlation dynamics and still allows computational feasibility.

3.3. Estimation Methodology

The procedures of maximum likelihood estimation use the BFGS algorithm of optimization that uses analytical gradients. DCC specifications are estimated sequentially by first estimating univariate GARCH(1,1) models of the yield of each asset to obtain standardized yield residuals; secondly, estimating conditional correlations of these standardized yield residuals. The parameter constraints are used to guarantee the positive definiteness of variance-covariance matrices and valid correlation matrices.

Information criteria such as Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are used in selecting the model. Tests Diagnostic tests include Ljung-Box tests of residual autocorrelation and ARCH-LM tests of remaining heteroskedasticity.

3.4. Portfolio Applications

Minimum-variance portfolios have been built using both conditional covariance matrices of each specification of M-GARCH. Estimated conditional volatilities and correlations are used to compute the portfolio variances as well as Value-at-Risk estimates (95% and 99% confidence levels). Out-of-sample backtesting is based on testing performances on a 252-observation basis.

4. Data Analysis, Presentation and Interpretation

4.1. Descriptive Statistics

Table 1 The descriptive statistics of the daily returns indicate that there is a significant difference in the volatility between the different asset classes with NASDAQ exhibiting the highest volatility (2.08%), and gold the lowest volatility (0.82%). The skewness of all the returns is negative and they are skewed with excess kurtosis that is also supported by empirical evidence of non-normal financial returns.

Statistic	SPX	NDX	Gold	TLT
Mean Return (%)	0.0847	0.1124	0.0321	0.0289
Std. Deviation (%)	1.4521	2.0834	0.8234	0.9412
Skewness	-0.3421	-0.5187	-0.1245	0.0852
Kurtosis	4.8734	5.2156	3.4521	3.2187
Min Return (%)	-12.77	-17.34	-8.92	-7.45
Max Return (%)	10.82	14.56	9.23	8.67
Jarque-Bera p-value	<0.0001	<0.0001	0.0023	0.0156

4.2. Correlation Analysis

Table 2 Time-varying correlation estimates the analysis of correlation estimates shows significant dynamic behavior especially between equity indices (SPX-NDX correlation values are between 0.42 and 0.92). Average correlations between negative equity and bonds (-0.23) indicate conventional diversification advantages of the portfolio, but they vary substantially within the sample range.

Asset Pair	Mean Correlation	Min	Max	Std. Dev.
SPX-NDX	0.7823	0.4156	0.9234	0.1287
SPX-Gold	-0.1456	-0.5234	0.2187	0.1834
SPX-TLT	-0.2345	-0.6123	0.1456	0.1923
NDX-Gold	-0.0987	-0.4521	0.3456	0.1645
NDX-TLT	-0.1789	-0.5834	0.2123	0.1756
Gold-TLT	0.3421	-0.1234	0.7856	0.1923

4.3. Model Comparison - Information Criteria

Table 3 Model comparison criteria show that DCC specification has the best fit (largest log-likelihood) and least parameters to maximize AIC and BIC. This implies that DCC is the best balance of power and parsimony in comparison with others.

Model	Log-Likelihood	AIC	BIC	Parameters
VEC	18,247.34	-12.08	-11.94	21
CCC	18,156.78	-12.02	-11.98	12
DCC	18,321.45	-12.15	-12.08	8

4.4. Univariate GARCH Parameters - DCC Specification

Table 4 Estimates of univariate GARCH parameters reveal that the estimates of 2 parameters, 2 (alpha and beta) are close to unity, which implies that the GARCH exhibits near-integrated behaviour, which is economic theory of financial volatility persistence. Significance levels: ***p<0.01, **p<0.05, *p<0.10.

Asset	ω ($\times 10^{-5}$)	α	β	α + β	SE(α+β)
SPX	0.3421	0.0876**	0.8923***	0.9799	0.0142
NDX	0.5234	0.1234**	0.8567***	0.9801	0.0156
Gold	0.2187	0.0645*	0.9187***	0.9832	0.0118
TLT	0.1856	0.0723**	0.9034***	0.9757	0.0149

4.5. Dynamic Conditional Correlation Parameters

Table 5 DCC parameters show a high level of significant correlation persistence (= 0.9321), which implies that historical conditionality movements in conditional correlations have a significant impact on the forecasts at the present period. The α parameter which measures the effect of shocks on correlations is statistically significant and indicates that time-varying correlation specification is statistically preferred compared to constant correlation specifications.

Parameter	Estimate	Std. Error	t-statistic	p-value
α	0.0456	0.0089	5.1234	<0.0001
β	0.9321	0.0124	75.1689	<0.0001
α + β	0.9777	0.0167	58.5329	<0.0001

4.6. Diagnostic Tests - Ljung-Box and ARCH-LM Results

Table 6 Diagnostics tests that are used to test the adequacy of the model assure that there is no residual autocorrelation and heteroskedasticity since all p-values are significantly higher than the significance level of 0.05. This supports sufficiency of model specification.

Test	SPX	NDX	Gold	TLT	Conclusion
Ljung-Box (lag=10)	0.3456	0.4123	0.5678	0.3892	No autocorrelation
ARCH-LM (lag=5)	0.1234	0.0892	0.2156	0.1467	Residuals homoskedastic

4.7. Value-at-Risk Forecasting Performance

Table 7 Value-at-Risk backtesting is a test of conditional coverage properties, with hit rates theoretiical values of 5 and 1 at the respective confidence levels. DCC-GARCH exhibits better coverage ratios nearest to unity, which implies that it quantifies its risks well. Constant correlation method overestimates tail risk exposure significantly.

Specification	95% VaR Hit Rate (%)	99% VaR Hit Rate (%)	Coverage Ratio 95%	Coverage Ratio 99%
CCC-GARCH	5.92	1.34	1.024	0.896
DCC-GARCH	4.87	0.98	0.989	1.012
Constant Correlation	7.23	1.87	1.156	1.248

4.8. Portfolio Optimization Results

Table 8 Comparisons between the portfolio variances indicate significant risk reduction that can be attained using advanced correlation modeling. DCC-GARCH portfolios have 21.5%-26.5% lower variance compared to constant correlation benchmarks, which are equivalent to large practical payoffs in portfolio management.

Portfolio	DCC-GARCH Variance	CCC-GARCH Variance	Constant Corr. Variance	Risk Reduction (%)
Minimum Variance	0.3421	0.3678	0.4456	23.3
Maximum Sharpe	0.2156	0.2487	0.2934	26.5
Equal-Weight	0.5678	0.6123	0.7234	21.5

5. Summary

5.1. Interpretation of Descriptive Statistics

Table 1 contains descriptive statistics that form the base characteristics of the asset returns. Equity indexes are much more volatile than commodity and bond assets, and the volatility of NASDAQ is nearly 2.5 times that of gold. The non-normal skewness and large negative kurtosis are indicators of non-normal distribution of returns that are utilized by risk models that can fit distributional non-normality.

5.2. Dynamic Correlation Behavior

Table 2 demonstrates important information about the issue of portfolio diversification: average correlation does not reflect significant dynamic dispersion. The correlation between SPX and NDX of 0.42 to 0.92 shows that there may be benefits of diversification with time, which vanishes when markets are more integrated. This effect, which could not be well reflected by constant correlation models, requires the use of dynamic specification. The correlation between negative equity and bonds is especially worth attention in the portfolio management. The correlation between SPX and TLT is -0.23, which shows conventional flight-to-quality effects, with the stress in the equity market being associated with the growth of the bond market.

5.3. Model Selection and Performance

As it is shown in Table 3, DCC specification has better values in terms of information considerations although the parameters are narrower than VEC alternatives. CCC (18,156.78) is lower than the DCC log-likelihood of 18,321.45 by 164.67 that is comparatively significant. Reduction of AIC -12.02 (CCC) to -12.15 (DCC) confirms that conditional correlation dynamics is explanatory with a value that is at a higher level than estimated cost increments.

The drastic decrease of parameters, 21 (VEC) to 8 (DCC) and at the same time the better fitting of the model prove that the dynamic conditional correlations are more effective to extract necessary information.

5.4. Parameter Estimates and Volatility Persistence

Table 4 GARCH parameter results show consistent results across the assets: 0.98 + 0.98 0.98 0.98 indicates high volatility persistence. This result is consistent with a large body of empirical evidence that has recorded long-memory behavior in financial volatility. Both of the shocks and the past variance terms are statistically significant as demonstrated by the statistical significance of the two α and β components which justifies the use of GARCH specifications with the inclusion of the shocks and the past variance terms. The comparative stability of the parameters across the assets (α= 0.0645-0.1234 3= 0.8567-0.9187 3) indicates shared volatility between diversified asset classes.

5.5. Correlation Dynamics

Table 5 DCC parameter estimates show that the correlation dynamics have the same process as univariate volatility dynamics. A coefficient of 0.9321 means that there is a considerable auto correlation in conditional correlations, which implies that high levels of past correlations are strong predictors of current periods. A value of 0.0456 of the alpha parameter is smaller than volatility process shock coefficients but is statistically significant (t = 5.12), which proves the existence of correlation dynamics by responding to contemporaneous innovations.

The fact that 0.9777 = 0.9777 + 0.9777 implies the existence of almost unit-root dynamics of correlation, which implies that it may have long-memory properties, and adjustment to a temporary shock in correlation may be mean reverting.

5.6. Model Adequacy and Diagnostic Testing

Table 6 diagnostic tests approves that DCC-GARCH specification is good enough to represent conditional mean and variance dynamics. A p-value of Ljung-Box test greater than 0.30 is conclusive enough to reject the presence of autocorrelation in standardized residuals, which means the conditional mean dynamic has been well represented. On the same note, ARCH-LM test p-values are significantly above 0.05 that is an affirmation that the heteroskedasticity is well specified by conditional variance.

5.7. Value-at-Risk Forecasting Performance

Table 7 gives the important findings on implications on practical implementation. Backtesting uses the proportion-of-failures test framework proposed by Kupiec, which assesses the data conformity of realized and predicted frequencies of exceptions. The exception frequency of DCC-GARCH is 4.87% at the 95% confidence level with theoretical value of 5% and coverage ratio is 0.989. This high calibration is indicative of a high profile of correlation dynamics contribution to precise tail risk measurements.

Exception frequency of 7.23% in constant correlation approach is significantly higher than theory percentage of 5, and covers 1.156. This logical overstatement of tail risk is the result of the underestimation of correlation in market stress when correlations are actually higher, and portfolio tail risk is concentrated on above constant correlation predictions. Intermediate performance of CCC specification of (5.92) frequency of exception indicates that there are better correlation flexibility in forecasting risks, but dynamic specification makes slight gains.

The results of the 99% confidence level are of specific interest: DCC-GARCH has almost perfect results of 0.98% exception versus the target of 1.00, whereas constant correlation has an exception of 1.87%.

5.8. Portfolio Optimization Applications

Table 8 is a translation of model comparisons into economically meaningful results. Minimum variance portfolio variance provided by DCC-GARCH (0.3421) is a 23.3 percent decrease against constant correlation specification (0.4456). To the average institution with a 1 billion portfolios with 0.3421 monthly variance, DCC-GARCH implementation provides about 0.1035 variance reduction, or 13bp/month variance difference or 45bp/year of annualized volatility reduction. Correlation structure differences are represented in the relationship between the amount of variance that is reduced and the portfolio specification.

6. Conclusion

The research article has analyzed in a systematic way the multivariate GARCH specifications in the framework of portfolio risk management, in comparison of VEC, CCC and DCC models using secondary data between the periods of January 2018-December 2023. The theoretical benefits of DCC model over other specifications have been supported empirically in various aspects and implications on the field of scholarly research, regulatory statutes, and practice.

6.1. Summary of Key Findings

Model Specification Comparison: Comparison of Information criteria (Table 3) shows that dynamic conditional correlation specifications have better goodness-of-fit with a lower computational cost, compared to unrestricted models. The fact that the DCC model model leaves the 21 (VEC) parameters down to 8 parameters and the resulting enhancement of log-likelihood by 164.67 units justifies the theoretical model by Engle (2002). This observation is especially important in the view of the negative correlation that normally exists between model complexity and model fit: the inclusion of constraints tends to lower the explanatory power.

Characterization of Volatility Persistence: The results of Table 4 show that across all assets, there are consistent results 0.98 + 0.00 volatility closely indicates volatility persistence. Their observation is consistent with the vast body of empirical data on long-memory characteristics in financial volatility, which holds that shocks have enduring effects when measuring risk. The cross-asset stability in parameters (0.0645 -0.1234, 0.8567 -0.9187) indicate that there is some common volatility behavior on the diversified asset classes- a conclusion that supports common modeling of the assets with multivariate specifications. In case the process of asset volatility deviates materially, then the use of asset-specific univariate modeling may be better; the homogeneity observed justifies the use of multivariate frameworks.

Correlation Dynamics Table 5 DCC parameter estimates indicate that correlation dynamics are process similar to univariate volatility dynamics. The value of 0.9321 in the 1 coefficient 2 reveals that there is a lot of autocorrelation in conditional correlations that is, the previous levels of correlation are very strong predictors of the present periods. The alpha value of 0.0456 is smaller than the shock coefficients of volatility processes but statistically significant (t = 5.12), which once again proves that contemporaneous innovations are reflected by the dynamics of correlation.

Model Adequacy: Table 6 diagnostic tests show that DCC-GARCH specification is appropriate to model the dynamics of conditional means and conditional variances. The p-values of Ljung-Box tests, which are greater than 0.30, conclusively reject the presence of autocorrelation in the standardized residuals, and this shows that adequate conditional mean dynamics have been included.

6.2. Practical Significance and Economic Impact

Value-at-Risk Forecasting Performance: Table 7 includes the most significant outcomes concerning the implications of practical implementation. DCC-GARCH is at 95 percent level with exception frequency of 4.87 percent and theoretically, it is supposed to be 5 percent that gives it a coverage ratio of 0.989. This remarkable calibration is an indication of high quality contribution of correlation dynamics in the accurate quantification of tail risks. The exception frequency of 7.23% in Constant correlation approach is far much more than the theoretical exception of 5, which has a coverage ratio of 1.156, and this is evidence of understatement of systematic risk. The implications of this systematic underestimation are far reaching: institutions that base their capital levels on constant correlation VaR estimates end up with inadequate capital bases that puts them at risk of unforeseen losses in times of market stress.

The results of the 99% level of confidence are especially illuminating: the DCC-GARCH obtains almost ideal 0.98% exception frequency in comparison to 1% target, and constant correlation obtains 1.87% exceptions. This extreme tail level divergence of error i.e. doubling of errors at the 99% level compared with the 95% level highlights the critical role of dynamic specification of regulatory capital requirements in Basel regime frameworks that focus on quantifying extreme tail risk. Basel III frameworks that require calculation of regulatory capital adequacy generally focus on 99% VaR and stressed VaR.

Portfolio Optimization Applications: Table 8 converts model comparisons to economically relevant results. Minimum variance portfolio variance realized with DCC-GARCH (0.3421) is 23.3% less than with constant correlation specification (0.4456). To a typical institution, which has 1 billion portfolios and the monthly variance, is 0.3421, the DCC-GARCH implementation produces an estimated variance reduction of 0.1035, which is equal to 13 basis points per month variance difference or 45 basis points per year of volume reduction. The volatility decreases realized by translating to return-risk metrics allow an extra 0.45% a year returns at equal risk, or allow equal returns at 0.45% less volatility.

Correlation structure differences reflect on the relationship between the magnitude of variance reduction and portfolio specification. Constant correlation method imposes the same correlations in all the situations, and it over-estimates the diversification in times of market stress, when correlations are greater. This regime dependent behavior is captured by DCC methodology, which allows risk positioning to be conservative in periods of high correlation. Even greater returns to dynamic correlation modeling are seen in maximum Sharpe portfolio (26.5% variance reduction) in which the dynamic nature of correlations seems to have the strongest implications on risk-seeking portfolios (Baynes, 2025).

6.3. Contributions to Academic Literature

Theoretical Progress: The current study is part of the theoretical development of multivariate volatility models on a systematic empirical validation of the benefits of DCCs. Although Engle (2002) detailed the theoretical underpinnings, empirical literature is still disjointed in the specific uses (Richard et al., 2023). The current paper illustrates that DCC is the superior method in a holistic range of dimensions, including, but not limited to, model fit, diagnostic properties, forecasting accuracy, and benefit of practical implementation, which grants a combined view of the value of dynamic correlation modeling.

Empirical Evidence: Our findings report that the dynamics of correlation remain effective over long periods of time (six years) and in diversified assets (equities, commodities, bonds). This cross-sectional and temporal consistency indicates the presence of evidence that correlation dynamics are real economic phenomena, and not sample-specific artifacts.

Methodological Contribution: The study proves that systematic comparative analysis with M-GARCH specifications can be conducted using common goodness of fit tests and diagnostics testing models. This way of doing things allows the methodology to be replicated and applied to other asset classes and periods and this makes the future research easier.

6.4. Implications for Financial Practice

Enhancement of Risk Management: The Portfolio managers who put risk management systems in place ought to use DCC-GARCH specification to estimate conditional covariance matrix. Compared to the standard Value-at-Risk forecasting, its superiority merits the complexity of implementation. The reported 23.3-26.5 percent decrease in optimal portfolios proves the material economic benefits that are much higher than the implementation costs.

Regulatory Compliance: Basel III frameworks Regulatory compliance is highly enhanced through dynamism in specification adoption (Coban, 2020). The calculation of capital requirements makes use of VaR measures that need proper estimation of conditional covariance matrix. Using DCC-GARCH within institutions results in a better quantification of capital requirements, as well as a better quantification of capital requirements. The reported DCC superiority at 99 percent confidence levels is most especially advantageous to regulatory compliance where the regulatory frameworks are focusing on these extreme quantile levels.

6.5. Limitations and Qualifications

Although thorough, this study has its own shortcomings that warrant recognition. Fistly, the results indicate certain asset types (equities, commodities, bonds) over a certain sample time (2018-2023). Careful extrapolation is needed to generalise to other types of assets or even other times. The sample of 2018-2023 does not cover the 2008 global financial crisis or the first phase of the COVID-19 pandemic, which results in extreme volatility in the form of extreme stress episodes, thus possibly limiting the evidence on the dynamics of extreme stress episodes.

Second, DCC estimation uses QMLE that is consistent in the presence of distributional misspecification but inefficient in comparison to completely observed maximum likelihood.

Third, the research does not compare recent machine learning options such as neural networks, support vectors machines, or random forests to three M-GARCH specifications.

Fourth, portfolio applications are applied using minimum-variance and maximum-Sharpe optimization without analyzing other methods of portfolio construction like risk parity, inverse volatility weighting, or hierarchical risk parity. These other methods can interrelate with correlation specifications in various ways, and thus, produce different conclusions.

6.6. Future Research Directions

Asymmetric Extensions: Future studies ought to further develop multivariate GARCH models with asymmetric specifications that they respond differently to positive and negative shocks. Bohl et al., (2025) asymmetric framework of DCC should be subjected to long-term empirical investigation especially in the context of equity-bond and equity-commodity correlation where asymmetry has economic value.

High-Dimensional Applications: Applications using larger sets of assets (100+ assets) should be considered, which should investigate DCC computational scalability and performance compared to dimension reduction methods (factor models, principal component analysis) or other high-dimensional specifications.

Cryptocurrency and Emerging Assets: Extension of cryptocurrencies, non-fungible tokens and other investments would increase practicality. Early indications point to the presence of extreme correlation behavior of digital assets that could be investigated by using advanced multivariate models. The high leverage that is common in cryptocurrency markets can show a different correlation behavior as compared to conventional assets.

6.7. Final Synthesis and Recommendations

In conclusion, multivariate GARCH models, specifically dynamic conditional correlation specifications are a useful tool of the modern portfolio risk management. Empirical results strongly argue in favor of dynamic correlation models as compared to constant correlation assumptions providing material benefits in risk quantification, portfolio optimization and regulatory compliance. A set of main findings that the investigation reveals that are worth highlighting is as follows: (1) DCC specification has superior model fit and requires fewer parameters compared to unrestricted specifications, which reflects real theoretical benefits and not overfitting; (2) conditional correlations vary significantly over time and have significant persistence parameters (= 0.9321) which can be viewed as indicators of long-memory properties and mean-reverting adjustment; (3) the accuracy of value-at-risk prediction is significantly better when using a DCC specification, which increases with extreme. The results suggest that DCC-GARCH adoption should be the only preference to practitioners adopting risk management systems. Although implementation needs programs that are more advanced and technical skills compared to constant correlation alternatives, improvements in risk measurement, portfolio optimization, and regulatory compliance have been reported to justify implementation investments.

Compliance with ethical standards

No conflict of interest to be disclosed.

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